

About the nonlocal particle-field matter

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Abstract

We propose that particles are associated both with localized macroscopic states at point vertices and with extended microscopic states at all vacuum points. The self-fields screen the microscopic particle currents everywhere except at the particle vertex, observed as a point mass/charge carrier of such a nonlocal particle-field object. All vertices are excluded from the microscopic Maxwell-Lorentz and Einstein-type equations for elementary objects with continuous particle densities. The geodesic particle motion depends on a vector force with unified electromagnetic and gravitational components. The advanced gravitational wave, which will never come from infinity at finite times, and the retarded electromagnetic wave are vector anti-waves. Curved pseudo-Riemannian space-time always maintains flat 3D space that is in agreement with the measurements of planetary perihelion precession, gravitational light bending, radar echo delay, and the nearly isotropic 2.73K cosmic microwave background. The developed synthesis of gravity with electrodynamics and the particle with its field corresponds to the predicted way of double unification and the Rainich-Misner criterion.

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1. Introduction

Pseudo-Riemannian geometry of curved space-time in General Relativity [1] is determined by the total system of gravitational masses in the Universe. Einstein's macroscopic relativity operates with one metric tensor for one curved space-time, which is considered as a common manifold for all bodies. Three-space is also curved in General Relativity due to Schwarzschild's solution for the point mass [2].

However, the balloon measurements [3] of the nearly isotropic 2.73K cosmic microwave background radiation strongly favour a spatial flatness of the Universe. Euclidean three-space may be accepted to modernize relativistic gravity, for example [4], and this space is quite appropriate for all other kinds of interactions. Having employed only spin-1 mediators (photon, W^\pm and Z mesons, and eight gluons), the Standard Model may anticipate the spin-1 graviton in the grand unification.

We consider different curved four-spaces for different elementary particles and their finite ensembles. And we try to justify Newtonian flat space and the absolute time rate due to intrinsic metric symmetries of these curved four-spaces. We accept that the proper pseudo-Riemannian four-space x_N^μ of one selected particle N with the mass m_N takes pseudo-Euclidean metrics (four-interval) in the absence of all other particles. Recall that space-time in General

Relativity is curved even for the sole mass m_N in the Universe. Metrics of the curved proper four-space x_N^μ , specified for one selected mass m_N , depends exclusively on a gravitational potential of external fields ($B_\mu^{\neq N}$ in our approach), with $ds_N^2(B_\mu^{\neq N}) = d\tau_N^2(B_\mu^{\neq N}) - dl_N^2$, $d\tau_N(B_\mu^{\neq N}) = [g_{oo}^N(dx_o + g_{Noo}^{-1}g_{oi}^N dx^i)]^{1/2}$, and $dl_N = [\gamma_{ij}^N dx^i dx^j]^{1/2}$, $i, j = 1, 2, 3$, and $\mu, \nu = 0, 1, 2, 3$. External systems are always different for different masses m_N and m_K , and metrics $ds_N(B_\mu^{\neq N})$ and $ds_K(B_\mu^{\neq K})$ of the pseudo-Riemannian four-spaces x_N^μ and x_K^μ , respectively, are different in principle if $N \neq K$.

A program goal of this paper is to derive electrodynamic and gravitational equations for one elementary particle N in its curved four-space x_N^μ and to sum these equations over an ensemble of particles. The world space x^i for this ensemble is introduced as a common intersection of the proper 3D-subspaces x_K^i which always maintain the universal (Euclidean) geometry within the proper curved four-spaces x_K^μ due to the symmetries $\gamma_{ij}^K \equiv g_{oi}^K g_{oj}^K (g_{oo}^K)^{-1} - g_{ij}^K = \delta_{ij}$.

At first we shall verify that the accepted tetrad formalism does not contradict to the intrinsic metric symmetries $\gamma_{ij}^K = \delta_{ij}$. Then we shall find the pseudo-Riemannian metric tensor $g_{\mu\nu}^K$, which satisfies to these symmetries under arbitrary gravitational and electromagnetic vector potentials and their gauges. This finding corresponds in principle to the existing opportunity to use the universal three-space x^i and co-ordinate time x^o as a common manifold $\{x^i; x^o\}$ for description of all particles and their fields. It is essential for our space and time notions that a 1D absolute time interval, $dt_K \equiv (\gamma_{oo}^K dx^o dx^o)^{1/2} = |dx^o| = dt > 0$, maintains also a universal form with $\gamma_{oo}^K = \delta_{oo}$ and may be applied to all elementary particles K.

The novel covariant scheme for relativity with flat 3D subspace is consistent with all known experiments and observations. In particularly, it explains quantitatively the measured planet perihelion precession, the gravitational light bending, and the radar echo delay. Our relativistic corrections to Newtonian motion in weak fields coincide with the similar corrections of General Relativity, but strong fields in our flat and GR's curved three-spaces lead to different dynamics of relativistic matter.

Our "non-curved" three-velocity $v_i \equiv \gamma_{ij} v^j = \delta_{ij} v^j$ and $v^i \equiv dx^i/d\tau_N$ in flat three-space admits a linear decomposition of a particle four-momentum, $P_{N\mu} = m_N V_\mu$ into an inertial, $m_N(1 - \delta_{ij} v^i v^j)^{-1/2} \{1; -\delta_{ij} v^j\}$, and field, $m_N(1 - \delta_{ij} v^i v^j)^{-1/2} \{\sqrt{g_{oo}} - 1; -\sqrt{g_{oo}} g_i\}$, summands that prohibited in General Relativity. And a linear superposition of gravitational, $m_N B_\mu(x)$, and electromagnetic, $q_N A_\mu(x)$, potentials takes place under flat space in the canonical four-momentum of the particle N with the charge q_N .

By following de Broglie's departure from the point classical particle, we put into consideration at all vacuum point $x \neq \xi_N, \xi_K$ the extended microscopic states of the particle N, located at one vertex point ξ_N^μ under its averaged, macroscopic state. These extended microscopic states may be formally considered as outgoing and incoming virtual fluctuations which induce locally outgoing and incoming elementary fields, $q_N f_{\mu\nu}^{N+}(x)_{x \neq \xi_N}$ and $-G m_N f_{\mu\nu}^{N-}(x)_{x \neq \xi_N}$, respectively, screening these particle fluctuations. A dual approach to the nonlocal particle notion, based on the extended microscopic and point macroscopic states, will allow us to exclude point vertices from the microscopic Maxwell-Lorentz and Einstein - type equations, but to hold point particles in the macroscopic equations. Our microscopic equations will manifest the local mass/charge/energy screening of the extended particle state by the induced field at every vacuum point, while our macroscopic equations will coincide with Maxwell's equations

and reveal the origin of the scalar Ricci curvature in the Einstein equation.

It is well known that retarded vector (spin-1) mediators can lead only to the repulsion of identical sources. But the extended, nonlocal particles allow us to employ advanced field potentials for the gravitational attraction of masses-outlets in the vector field theory. In other words, we relate the retarded potentials $q_N a_{N\mu}^+$ with outgoing electromagnetic fields and advanced potentials $-Gm_N a_{N\mu}^-$ with incoming gravitational fields in the Maxwell-type equations. These elementary potentials contribute to the total gravitational, $B_\mu = -\sum_K^{all} Gm_K a_{K\mu}^-$, and electromagnetic, $A_\mu = \sum_K^{all} q_K a_{K\mu}^+$, field potentials.

The unified vector structure of gravitational and electromagnetic forces, acting on the particle, allows to generalize the equivalence principle on the proper canonical four-space. The geodesic equation, $DP_{N\mu}(\xi_N) = 0$, for the charged vertex incorporates the retarded Lorentz and the advanced gravitational forces. The vertex charge q_N is always a source of the outgoing electromagnetic field, while the vertex mass m_N is always an outlet of the incoming gravitational field. The outgoing (from the point vertex) electromagnetic wave and the incoming (from infinity) gravitational wave are coupled vector anti-waves of each other. These waves never cross at finite times and they are not relevant to metric modulations of three-space, prohibited in the present scheme. The strict spatial flatness of the Universe on micro-, macro-, and mega-scales is the principle point of our relativistic scheme.

The global superposition of all microscopic particle states is considered in the last section, where we derive the macroscopic gravitational equations. The scalar Ricci curvature is absent in the microscopic equations, but appears naturally in the Einstein macroscopic equation in agreement with the Rainich-Misner criterion. The known criterion of double unification, the particle with its fields and gravity with electrodynamics, is also satisfied in the developed theory with nonlocal elementary objects.

2. Flat three-space and absolute time

In order to verify the mathematical opportunity to implicate the flat 3D subspace x_N^i into the proper four-space x_N^μ with curved pseudo-Riemannian metric under consideration of one elementary mass N, we employ the known tetrad formalism, for example [5,6], which represents a proper four-interval, $ds_N^2 \equiv g_{\mu\nu}^N dx_N^\mu dx_N^\nu \equiv ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu \equiv \eta_{\alpha\beta} e_\mu^\alpha e_\nu^\beta dx^\mu dx^\nu \equiv \eta_{\alpha\beta} dx^\alpha dx^\beta$, in the "plane" coordinates $dx^\alpha \equiv e_\mu^\alpha dx^\mu$, $dx^\beta \equiv e_\nu^\beta dx^\nu$, $\eta_{\alpha\beta} = \text{diag}(+1, -1, -1, -1)$. One can promptly find $e_\mu^o = \{\sqrt{g_{oo}}; -\sqrt{g_{oo}g_i}\}$ and $e_\mu^a = \{0, e_i^a\}$ from the equality $ds^2 \equiv [\sqrt{g_{oo}}(dx^o - g_i dx^i)]^2 - \gamma_{ij} dx^i dx^j$, $g_i \equiv -g_{oi}/g_{oo}$. At first glance the proper spatial triad $e_i^a \equiv e_{N_i}^a$ ($a = 1, 2, 3$, while $\alpha = 0, 1, 2, 3$) depends essentially on gravitational fields of external masses because this triad can be represented via all components of the proper metric tensor $g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} \equiv \gamma_{ij} \equiv \gamma_{ij}^N$. But this is not a case if there are intrinsic symmetries of the proper metric tensor $g_{\mu\nu}^N$.

Let us consider space components V_i of the metric-velocity four-vector $V_\mu \equiv g_{\mu\nu} dx^\nu/ds$ by using the tetrad formalism, $-(\sqrt{g_{oo}g_i} + v_i)(1 - v_i v^i)^{-1/2} = V_i = e_i^\alpha V_\alpha = e_i^o V_o + e_i^a V_a = -(\sqrt{g_{oo}g_i} + e_i^a v_a)(1 - v_a v^a)^{-1/2}$. This leads immediately to trivial relations $v_i \equiv e_i^a v_a = \delta_i^a v_a$ between the "curved", $v_i = \gamma_{ij} dx^j / \sqrt{g_{oo}} |dx^o - g_i dx^i|$, and the "plane", v_a , three-velocities because $e_i^o =$

$-\sqrt{g_{oo}}g_i$ and $V_\alpha = \{(1-v_av^a)^{-1/2}; -v_a(1-v_av^a)^{-1/2}\}$. These relations through the universal Kronecker delta-symbols δ_i^a clearly indicate that all spatial triads and, consequently, the three-space metric tensor are irrelevant to gravitation fields, *i.e.* $e_{Ni}^a = \delta_i^a$ and $\gamma_{ij}^N = \delta_{ij}$. Below we introduce the metric tensor $g_{\mu\nu}^N$ which satisfy these symmetries in all field potentials and their gauges.

Again, Euclidean spatial geometry takes place in the covariant formalism for gravitation due to the intrinsic metric symmetries, $g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} \equiv \delta_{ij}$, of proper four-spaces of elementary particles. The four-space metric tensor in the most general case reads $g_{\mu\nu} \equiv \eta_{\alpha\beta}e_\mu^\alpha e_\nu^\beta \equiv \eta_{\mu\nu} + \eta_{\alpha\beta}(e_\mu^\alpha e_\nu^\beta - \delta_\mu^\alpha \delta_\nu^\beta)$, where $e_\mu^o = \{\sqrt{g_{oo}}; -\sqrt{g_{oo}}g_i\}$ and $e_\mu^a = \{0, \delta_i^a\} \equiv \delta_\mu^a$. In agreement with this consideration, all three-intervals dl_K are always associated with the universal Euclidean metrics, because $\gamma_{ij}^K \equiv g_{oi}g_{oj}(g_{oo}^K)^{-1} - g_{ij}^K \equiv \delta_{ij} \equiv -\eta_{ij}$ for all particles, while different proper four-intervals ds_K represent different pseudo-Riemannian metrics, because $g_{\mu\nu}^N \neq g_{\mu\nu}^K$.

A proper four-interval of one selected mass N ($ds_N \equiv ds$ and $dx_N \equiv dx$, for short) is given by

$$ds^2 = \left(\sqrt{g_{oo}}dx^o + \frac{g_{oi}dx^i}{\sqrt{g_{oo}}} \right)^2 - \delta_{ij}dx^i dx^j \equiv [d\tau(s)]^2 - dl^2 \quad (1)$$

in arbitrary external gravitational fields. It is worth noting that (1) is a very complicated nonlinear equation with respect to the proper metrics ds . The proper time $d\tau \equiv d\tau_N(s_N) \equiv \sqrt{g_{oo}}|dx^o - g_i dx^i|$ in (1) depends on $|ds| = \sqrt{[d\tau(s)]^2 - dl^2}$, and the nonlinear four-interval ds depends on the 3D interval $dl \equiv dl_N \equiv \sqrt{\delta_{ij}dx^i dx^j} > 0$ in a nontrivial way for moving bodies.

The co-ordinate time rate dx_N^o of the advanced and retarded field matter has opposite directions resulting to incoming and outgoing material fields. We can use the absolute time interval, $dt \equiv dt_N \equiv \sqrt{\gamma_{oo}^N dx_N^o dx_N^o} \equiv \sqrt{\delta_{oo} dx_N^o dx_N^o} \equiv |dx_N^o| > 0$, for both outgoing (retarded) and incoming (advanced) fields. Both the 3D space interval dl_N and the 1D time interval dt_N are based on the universal metric symbols, $\gamma_{ij}^N = \delta_{ij}$ and $\gamma_{oo}^N = \delta_{oo}$, respectively. But there is no flat, homogeneous metrics $ds_N \equiv (\text{sign } dx_N^o) \sqrt{g_{\mu\nu}^N(x_N) dx_N^\mu dx_N^\nu}$ of four-space, because $g_{\mu\nu}^N(x_N) \neq \text{const.}$ The absolute or flat "rulers" dl_K and dt_K are relevant in practice for measurements and observations, and universal (Euclidean) geometry of 3D and 1D proper subspaces ought to stand behind the world (common) three space and the world (common) time notions. Flat space and flat time, *i.e.* the flat world space+time $\{x^i; x^o\}$, can always be applied to all particles and fields in the Universe, while $\{x^\mu\}$ and ds_N is to be specified for every selected particle N (or for their finite ensemble).

Now we study the metric-velocity four-vector V_μ . Notice that $V_\mu = e_\mu^\alpha V_\alpha = (e_\mu^a V_a + e_\mu^o V_o) = (e_\mu^a V_a + \delta_\mu^o V_o) + (e_\mu^o - \delta_\mu^o) V_o \equiv \mathcal{V}_\mu + U_\mu$, with the proper four-velocity $\mathcal{V}_\mu \equiv (e_\mu^a V_a + \delta_\mu^o V_o) = \delta_\mu^a V_a$, because $e_o^a = 0$ and $e_i^a = \delta_i^a$. Flat three-space geometry is just a way to introduce the four-potential $U_\mu \equiv (e_\mu^o - \delta_\mu^o) V_o = B_\mu + (q_N/m_N)A_\mu + \partial_\mu \phi_N$ for both gravitational, B_μ , and electromagnetic, $(q_N/m_N)A_\mu$, components. Let a total (canonical) particle four-momentum $P_{N\mu}(x)$ has formally the inertial, gravitational and electromagnetic contributions,

$$\begin{aligned} P_{N\mu}(x) &\equiv m_N \frac{g_{\mu\nu}^N dx_N^\nu}{ds_N} = \left\{ \frac{m_N}{\sqrt{1-v^2}} + \frac{m_N(\sqrt{g_{oo}}-1)}{\sqrt{1-v^2}}; -\frac{m_N v_i}{\sqrt{1-v^2}} - \frac{m_N g_i \sqrt{g_{oo}}}{\sqrt{1-v^2}} \right\} \\ &= m_N V_{N\mu} = m_N (\mathcal{V}_\mu + B_\mu + q_N m_N^{-1} A_\mu + \partial_\mu \phi_N) = m_N (\mathcal{V}_\mu + B_\mu^{\neq N} + S_\mu^N), \quad (2) \end{aligned}$$

where $v_i \equiv \gamma_{ij}v^j$, $v^2 \equiv v_i v^i$, $|ds| = (dx_\mu dx^\mu)^{1/2}$, $dx_\mu = g_{\mu\nu}dx^\nu$, $dx^\mu \equiv dx^\mu_N$, $v^i \equiv dx^i/g_{oo}^{1/2}(dx^o - g_i dx^i)$; $g_i = -g_{oi}/g_{oo}$; $\gamma_{ij} \equiv g_i g_j g_{oo} - g_{ij} = \delta_{ij} = -\eta_{ij}$.

Flat three-space admits a direct analogy in structures of gravitational and electromagnetic fields (which will be proved below). Namely, the gravitational, $B_\mu(x) \equiv -\sum_K^{all} Gm_K a_{K\mu}^-(x)$, and the electromagnetic, $A_\mu(x) \equiv \sum_K^{all} q_K a_{K\mu}^+(x)$, potentials in the canonical four-momentum (2) are based on the elementary potentials $-Gm_K a_{K\mu}^-(x)$ and $q_K a_{K\mu}^+(x)$, respectively, with coupled forming-up pre-potentials $a_{K\mu}^\pm(x)$. The latter are a gauge family of advanced (-) and retarded (+) field solutions, $a_{K\mu}^\mp(x) = \tilde{a}_{K\mu}^\mp(x) + \partial_\mu \chi_K^\mp(x)$, of the Maxwell-type equations (derived below) for the mass m_K and the charge q_K , respectively.

Finally the canonical four-momentum $P_{N\mu}(x)$ depends on the inertial momentum $m_N \mathcal{V}_{N\mu}(x)$, the "internal" self-momentum $m_N S_\mu^N(x) \equiv q_N A_\mu(x) - Gm_N^2 a_{N\mu}^-(x) + m_N \partial_\mu \phi_N(x)$, and the "external" field momentum $m_N B_\mu^{\neq N}(x) \equiv -m_N \sum_K^{\neq N} Gm_K a_{K\mu}^-(x)$ in the most general case.

The "internal" self-potential $S_\mu^N(x)$ depends on the proper gauge, $\partial_\mu \phi_N$, and the particle parameters, m_N and q_N , while the "external", gravitational potential is independent from the proper particle parameters. A physical gauge can assist to avoid both a self-action in the proper four-interval ds_N and an infinite energy of the self-field N at the vertex point ξ_N .

At first we study the relations (2) for an electrically uncharged particle, when $q_N = 0$ and $S_\mu^N(x) = \partial_\mu \phi_N - Gm_N a_{N\mu}^-(x)$. Notice from the relations (2) that the passive gravitational mass in $m_N B_\mu^{\neq N}$ is always equal to the inertial (rest) mass in $m_N \mathcal{V}_\mu$. Contrary to General Relativity, the electrodynamics-like gravitation (based on the linear superposition of elementary fields) may propose from (2) a detail structure of the proper metric tensor $g_{\mu\nu}^N(x)$. The proper tetrad takes the following components $e_\mu^a = \{0, \delta_i^a\} = \delta_\mu^a$ and $e_\mu^o = \{1 + \sqrt{1-v^2}(B_o + \partial_o \phi_N); \sqrt{1-v^2}(B_i + \partial_o \phi_N)\} = \delta_\mu^o + \sqrt{1-v^2}(B_\mu + \partial_\mu \phi_N)$. Than the proper metric tensor $g_{\mu\nu}^N \equiv g_{\mu\nu}^N(x_N)$ for the selected mass m_N is given by

$$\begin{cases} g_{oo}^N = [1 + \sqrt{1-v^2}(B_o^{\neq N} + S_o^N)]^2 \\ g_{oi}^N = [1 + \sqrt{1-v^2}(B_o^{\neq N} + S_o^N)]\sqrt{1-v^2}(B_i^{\neq N} + S_i^N) \\ g_{ij}^N = (1-v^2)(B_i^{\neq N} + S_i^N)(B_j^{\neq N} + S_j^N) - \delta_{ij} \end{cases} \quad (3)$$

Notice that the proper gravitational potential, $\mathcal{B}_\mu^N(x) \equiv -Gm_N a_{N\mu}^-(x) \neq 0$ in (3), does not vanish in the absence of external gravitational fields, when $B_\mu^{\neq N} = 0$. This proper potential should take the Newtonian form in the particle rest frame, with $B_o^N(x) = |\mathbf{x} - \boldsymbol{\xi}_N|^{-1} \rightarrow \infty$ when $|\mathbf{x} - \boldsymbol{\xi}_N| \rightarrow 0$. An infinite value of such proper potential, $\mathcal{A}_\mu^N(x) \equiv q_N a_{N\mu}^+(x)$, of the point electric charge q_N at its path points ξ_N is conventionally omitted in Classical Electrodynamics. The similar divergence problem may be resolved more appropriately in (2)-(3) due to a physical choice of the gauge term $\partial_\mu \phi_N$, when $S_\mu^N(x)P_N^\mu(x) = [\partial_\mu \phi_N - Gm_N a_{N\mu}^-(x, \xi_N)]P_N^\mu(x) = 0$. In other words, the internal self-momentum behaves like an internal angular momentum or a particle spin with $S_\mu^N(x) = \{0; \mathbf{S}(x)\}$ in the rest frame of references.

As it was expected, all components of the three-space metric tensor are independent from gravitational fields. Now the equality $\gamma_{ij} \equiv g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} = \delta_{ij}$ may be verified directly from (3) despite every component of the metric tensor $g_{\mu\nu}$ depends on the field potentials. One can represent (3) in a more compact way, $g_{\mu\nu} = \eta_{\alpha\beta}e_\mu^\alpha e_\nu^\beta$ and $e_\mu^\alpha = \delta_\mu^\alpha + \delta^{\alpha o}\sqrt{1-v^2}B'_\mu$. Notice, that $P_N^\mu = g^{\mu\nu}P_{N\nu} = m_N[\eta^{\mu\nu}\mathcal{V}_\nu + (g^{\mu\nu} - \eta^{\mu\nu})\mathcal{V}_\nu + g^{\mu\nu}B'_\nu] = m_N\eta^{\mu\nu}\mathcal{V}_\nu - m_N\{(1 +$

$\sqrt{1-v^2}B'_o)^{-1}(B'_o + B'_i v^i); 0\} = m_N(1-v^2)^{-1/2}\{1 + (g_{oo}^{-1/2} - 1 + g_i v^i); v^i\}$ and $P_{N\mu}P_N^\mu = m_N^2(\mathcal{V}_\mu\mathcal{V}^\mu + 0) = m_N^2$ for an arbitrary gravitational four-potential $B'_N \equiv B_\mu^{\neq N} + S_\mu^N = B_\mu + \partial_\mu\phi_N$, when $q_N = 0$.

Substituting the metric tensor (3) into (1), we can obtain a general equation for the proper four-interval, $ds = ds_N$, of one selected mass m_N ,

$$ds^2 + dl^2 = (dx^o + \sqrt{1-v^2}B'_\mu V^\mu ds)^2, \quad (4)$$

with $dl^2 \equiv \delta_{ij}dx^i dx^j$. From here the proper particle time $d\tau$ and the particle four-interval ds are not self-affected by the particle mass and charge only under the physical gauge $S_\mu^N P_N^\mu \equiv m_N S_\mu^N V_N^\mu = 0$, when $B'_\mu V^\mu = (B_\mu^{\neq N} + S_\mu^N)V^\mu = B_\mu^{\neq N} V^\mu$.

Each elementary gravitational potential $-Gm_K a_{K\mu}^-(x)$ in $B_\mu^{\neq N}(x)$ is related to its distant point vertex ξ_K in agreement with the Maxwell-type equation (derived below). This potential in the two-body problem has only Newton's component, $B_\mu^{\neq N} = -Gm_K a_{K\mu}^-(x) = \{-Gm_K |\mathbf{x} - \xi_K|^{-1}; 0\}$, for a point mass m_N at $x = \xi_N$ in the mass m_K frame of references. Below we use this potential to study gravitational phenomena in the Sun's field.

3. Relativity tests in the Sun's field

3.1. The planetary perihelion precession in the flat world space

Now we derive the planetary perihelion precession in order to test in practice the four-interval equations (4) with the novel structure of the metric tensor (3), which is consistent with flat three-space, $\gamma_{ij} = \delta_{ij}$. A distant system of elementary gravitational centres K may be considered as one united center with the joint mass (the Sun, for example, with the Sun's mass M_S), when all $|\mathbf{x} - \xi_K| \approx r \equiv u^{-1}$. Nonlinear relativistic relations for the function $\alpha_N(ds^2, dl^2) = -\sqrt{1-v_N^2}B_\mu^{\neq N}V_N^\mu$ will be derived in the next section. One may use in (4) the Newton potential, $(-B_o^{\neq N}) = GM_S r^{-1} \equiv \mu u \ll 1$, for the non-relativistic motion when a considered mass N (a planet with mass $m_N \ll M_S$ and $v_N^2 \equiv dl^2/d\tau^2 \ll 1$) moves in the Sun's rest frame, where $B_i^{\neq N} = 0$. The equation (4) for the proper four-interval ds reads

$$\begin{aligned} ds^2(t, l, \tau) + dl^2 &\equiv d\tau^2(t, l, s) = dt^2 \left(1 - \mu u \sqrt{1 - \frac{dl^2}{d\tau^2}}\right)^2 \\ &\approx dt^2 \left((1 - \mu u)^2 + \mu u(1 - \mu u) \frac{dl^2}{d\tau^2}\right) \approx (1 - 2\mu u)dt^2 + \mu u dl^2, \end{aligned} \quad (5)$$

where we used $dt \equiv |dx^o|$, $\mu u \ll 1$, $dl^2 \ll d\tau^2$, and $dt^2 - d\tau^2 \ll dt^2$.

The mass dependent coefficient $1 - \mu u$ in (5) at the flat three-interval, $dl^2 = dr^2 + r^2 d\varphi^2 = u^{-4} du^2 + u^{-2} d\varphi^2$, does not mean departure from Euclidean spatial geometry in gravitational fields. This coefficient is related to a direct involvement of the space replacement dl into the proper time $d\tau(dt, dl, ds)$. Our proper time in (5), $d\tau \approx (1 - 2\mu u + \mu u v^2)^{1/2} dt$, coincides with the General Relativity proper time, $(1 - 2\mu u)^{1/2} dt$, in the limit of weak fields and small velocities.

The Killing vectors and integrals of motion, $(1 - \mu u)^2 dt/ds = E = \text{const}$ and $r^2 d\varphi/ds = L = \text{const}$ (with $\vartheta = \pi/2 = \text{const}$), are well known under the four-interval with stationary coefficients, for example [7]. By taking into account

these conservation laws in (5), one obtains an equation for a rosette motion of planets under the non-relativistic restrictions on their velocities,

$$(1 - 2\mu u)L^{-2} + (1 - 3\mu u)(u'^2 + u^2) = E^2 L^{-2}, \quad (6)$$

where $u' \equiv du/d\varphi$ and $\mu u \ll 1$. Now one may differentiate (6) with respect to the polar angle φ ,

$$u'' + u - \frac{\mu}{L^2} = \frac{9}{2}\mu u^2 + 3\mu u''u + \frac{3}{2}\mu u'^2, \quad (7)$$

by keeping only the oldest nonlinear terms. This equation may be solved in two steps, when a linear solution, $u_o = \mu L^{-2}(1 + \epsilon \cos\varphi)$, is substituted into the nonlinear terms on the right hand side of (7).

The most important correction (which is summed over century rotations of the planets) is related to the "resonance" (proportional to $\epsilon \cos\varphi$) nonlinear terms. We therefore ignore in (7) all corrections, apart from $u^2 \sim 2\mu^2 L^{-4} \epsilon \cos\varphi$ and $u''u \sim -\mu^2 L^{-4} \epsilon \cos\varphi$. Then the approximate equation for the rosette motion, $u'' + u - \mu L^{-2} \approx 6\mu^3 L^{-4} \epsilon \cos\varphi$, leads to the well known perihelion precession, $\Delta\varphi = 6\pi\mu^2 L^{-2} \equiv 6\pi\mu/a(1 - \epsilon^2)$, which also may be derived through Schwarzschild's metric formalism with curved three-space, for example [5-10].

It is important to emphasize for a verification of our space concept that the observed result for the planet perihelion precession $\Delta\varphi$ in the Sun's field was derived from the GR nonlinear four-interval (5) under flat three-space, rather than from the linear four-interval under curved three-space.

3.2. The radar echo delay in flat three-space

The gravitational redshift of light frequency ω is still considered in some textbooks as a confirmation of the accepted opinion that gravity couples to the energy content of any matter, including the photon's energy E_γ or the "relativistic mass" $m_\gamma = E_\gamma/c^2$. We agree completely with the statement $E = mc^2$ for rest-mass particles, but disagree with its inverse reading, $m = E/c^2$, for all kinds of matter, in particular for photons with $m = 0$ and $E \neq 0$.

In 1907 Einstein introduced the principle of equivalence for a uniformly accelerated body and concluded that its potential energy is associated with the "heavy" (passive) gravitational mass [11]. This Einstein's conclusion was formally generalized in a way that any energy, including light, has a gravitational mass. Proponents of this generalization assume that photon's energy-"relativistic mass" is attracted directly by the Earth in agreement with the measured redshift $\Delta\omega/\omega = \Delta E_\gamma/E_\gamma = \Delta(-m_\gamma G M_E R_E^{-1})/m_\gamma c^2 = -\Delta G M_E/R_E c^2$ due to these formal relations for light in the static gravitational field. But the formal introduction of the "relativistic mass" for matter with a zero scalar mass invariant leads to the underestimated light deflection, $\varphi = -2GM_S/R_S c^2 \equiv -2\mu/c^2 R_S \equiv -r_g/R_S$, under the Newtonian "fall" of photons in the Sun's gravitational field [12]. In 1917, when the Schwarzschild's solution [2] for space curvature had been accepted by General Relativity, Einstein predicted the non-Newtonian light deflection $\varphi = -2r_g/R_S$, based once again on a direct attraction of the photon's energy E_γ by the Sun. All experimental tests supported later the corrected Einstein's result for the gravitational light deflection, that up till now holds non-Euclidean three-space in contemporary developments of General Relativity.

But do photon's gravitational phenomena undoubtedly confirm that space is really curved and gravity couples to the photon's energy? The principle of equivalence was proposed only for a rest-mass body with a proper reference system [11], while the photon has no inertial or rest mass at all. How may a passive gravitational mass be attributed to a particle without an inertial mass?

Now we revise the conventional theory of the radar echo delay and the gravitational light deflection. Our goal is to show that the known measurements [9,13,14] of light phenomena in the Solar system may be interpreted as confirmations of the world space flatness. Let us consider a static ($g_i = 0$, for simplicity) gravitation field, where the physical slowness of photons, $n^{-1} \equiv v/c$, can be derived directly from the covariant Maxwell equations [5], $n^{-1} = \sqrt{\epsilon\mu} = \sqrt{g_{oo}}$. The measured, physical velocity $v = dl/d\tau$, as well as the measured, physical frequency $\omega = \omega_o dt/d\tau$, is to be specified with respect to the physical time $d\tau = \sqrt{g_{oo}}dt$. Originally Einstein associated the light's redshift with the different clock rates in the Sun's gravitational potential [11], and this true nature of the redshift is irrelevant to the energy content or to the "apparent weight" [13] of the massless photon. In other words, the light redshift is simply the gravitational blue shift of clocks which measure the photon frequency.

As compared to the physical velocity of light, its co-ordinate velocity $v = dl/dt$ is double shifted by the static gravitational potential $\sqrt{g_{oo}}$,

$$i \equiv \frac{dl}{dt} \equiv \frac{dl}{d\tau} \frac{d\tau}{dt} = cn^{-1} \sqrt{g_{oo}} = cg_{oo} = c \left(1 - \frac{r_g}{2r}\right)^2, \quad (8)$$

in the Sun's gravitational field, where $r_g \equiv 2GM_S/c^2 = 2,95 \text{ km}$, $r_g/r \ll 1$. Notice that the local physical slowness $n^{-1} = \sqrt{g_{oo}}$, and the local time dilation $d\tau/dt \equiv \sqrt{g_{oo}}|1 - g_i \dot{x}^i| \approx \sqrt{g_{oo}}$ are responsible together for the double co-ordinate slowness in the relation (8).

A world time delay of Mercury's radar echo reads through the co-ordinate relation (8) as

$$\Delta t = 2 \int_{l_E}^{l_M} dl \left(\frac{1}{i} - \frac{1}{c} \right) \approx \frac{2}{c} \int_{x_E}^{x_M} \frac{r_g dx}{\sqrt{x^2 + R_S^2}} \approx \frac{2r_g}{c} \ln \frac{4r_{MS}r_{ES}}{R_S^2} = 220 \mu s, \quad (9)$$

where $y \approx R_S = 0.7 \times 10^6 \text{ km}$ is the radius of the Sun, while $r_{ES} = 149.5 \times 10^6 \text{ km}$ and $r_{MS} = 57.9 \times 10^6 \text{ km}$ are the Earth-Sun and Mercury-Sun distances, respectively. It is essential that we use Euclidean metrics for any finite distance, $r = (x^2 + y^2)^{1/2}$, between the Sun's center (0,0) and any considered point (x, y) on the photonic axis x. One can measure in the Earth's laboratory only the physical time delay $\Delta\tau_E$, which practically coincides with the absolute world time delay Δt in the Earth's weak field, *i.e.* $\Delta\tau_E \approx \Delta t = 220 \mu s$. The known experimental results [14] correspond to the radar echo delay (9), based on the concept of flat world space.

3.3. The gravitational light bending in flat three-space

A co-ordinate angular deflection φ of a light front in the Sun's gravitational field may be derived geometrically by using the co-ordinate velocity (8),

$$\varphi_\infty = -2 \int_0^\infty dl \frac{\partial}{\partial y} \left(\frac{i}{c} \right) \approx -2 \int_0^\infty dx \frac{\partial}{\partial y} \left(\frac{r_g}{\sqrt{x^2 + y^2}} \right)$$

$$\approx -2r_g \int_0^\infty dx \frac{R_s}{(x^2 + R_s^2)^{3/2}} = -\frac{2r_g}{R_s} = -1.75''. \quad (10)$$

A physical way to derive the ray deflection (10) is to apply Fermat's principle to light in gravitational fields. We relate the covariant wave vector K_o in the scalar equation $K_\mu K^\mu = 0$ to the measured, physical energy-frequency of the photon ($K_o = E = \hbar\omega = \hbar\omega_o/\sqrt{g_{oo}}$, $\hbar\omega_o = \text{const} > 0$; notice that P_o is also the measured particle's energy in $P_\mu P^\mu = m^2$) in agreement with the original Einstein's paper [11].

The scalar wave equation $K_\mu K^\mu = g_N^{\mu\nu} K_\mu K_\nu = 0$ has two solutions for paired anti-waves. One solution for the electromagnetic wave (photon), with

$$\begin{cases} K_o \equiv \hbar\omega_o dt/cd\tau = g_{oo}(K^o - g_i K^i) \\ \gamma_{ij} K^i K^j = g_{oo}(K^o - g_i K^i)^2 = K_o^2/g_{oo} = \hbar^2 \omega_o^2 dt^2/c^2 g_{oo} d\tau^2 \\ K^i = \hbar\omega_o dt dx^i/c\sqrt{g_{oo}} d\tau dl, K_i = -(\hbar\omega_o dt/c\sqrt{g_{oo}} d\tau)[(\gamma_{ij} dx^j/dl) + \sqrt{g_{oo}} g_i] \end{cases}, \quad (11)$$

or $K_\mu = \{+E, \mp E[(\delta_{ij} dx^i/\sqrt{g_{oo}} dl) + g_i]\}$, and another solution for the gravitational wave (graviton) $G_\mu \equiv -K_\mu = \{-E, \pm E[(\delta_{ij} dx^i/\sqrt{g_{oo}} dl) + g_i]\}$, with $G_\mu G^\mu = 0$. The graviton, the anti-photon in our consideration, is associated formally with the negative energy $G_o = -E < 0$ (like the negative-energy right-handed neutrino in the Weyl equation) or with the backward (advanced) wave motion under the positive world time direction, $dt = |dx^o| > 0$. By employing one absolute time rate $dt > 0$ for measurements, one should relate gravity with the advanced, incoming material fields moving from infinity to their point outlets ξ_K^i in flat world space x^i .

The Fermat variations with respect to $\delta\varphi$ and δu ($r \equiv u^{-1}$, φ , and $\vartheta = \pi/2$ are the spherical coordinates) for photons in a static gravitational field,

$$\delta \int K_i dx^i = -\delta \int \frac{\hbar\omega_o \gamma_{ij} dx^j}{cg_{oo} dl} dx^i = -\frac{\hbar\omega_o}{c} \delta \int \frac{\sqrt{du^2 + u^2 d\varphi^2}}{u^2(1 - 2^{-1}r_g u)^2} = 0, \quad (12)$$

where $\gamma_{ij} = \delta_{ij}$ and $dl = \sqrt{\delta_{ij} x^i x^j} = \sqrt{dr^2 + r^2 d\varphi^2}$ is the Euclidean three-interval, leads to a couple of light ray equations,

$$\begin{cases} (1 - 2^{-1}r_g u)^4 [(u'_\varphi)^2 + u^2] = U_o^2 = \text{const} \\ u''_{\varphi\varphi} + u = U_o^2 r_g (1 - 2^{-1}r_g u)^{-5} \approx U_o^2 r_g \end{cases}. \quad (13)$$

Solutions of (13), $u \equiv r^{-1} = r_o^{-1} \sin\varphi + r_g r_o^{-2} (1 + \cos\varphi)$ and $r_g/r_o \approx r_g U_o \approx r_g/R_s \ll 1$, may be used under the Sun's weak field. The propagation of light from $r(-\infty) = \infty, \varphi(-\infty) = \pi$ to $r(+\infty) \rightarrow \infty, \varphi(+\infty) \rightarrow \varphi_\infty$ corresponds to the angular deflection $\varphi_\infty = \arcsin[-2r_g R_s^{-1} (1 + \cos\varphi_\infty)] \approx -2r_g/R_s$ from the initial light's direction. This deflection coincides with (10) and is in agreement with the known measurements $-1,66'' \pm 0.18''$, for example [9].

The massless photon is not attracted directly by the Sun, but exhibits the physical velocity slowness $v/c \equiv dl/cd\tau = \sqrt{g_{oo}}$, with $g_{oo} < 1$ and $\gamma_{ij} = \delta_{ij}$, under locally dilated time rate. Both the photon and the graviton have neither inertial nor gravitational masses. Massless gravitons are not coupled to each other, and gravitational interactions, like electrodynamic ones, are linear in the present theory of the flat world space with the absolute time rate. The mass four-current is the origin of the vector gravitational field according to the Maxwell-type equation (derived below). This nature of gravity is in agreement with the zero measurements of the Nordtvedt effect [15,16] and satisfies the Einstein

principle of equivalence. Both the vector gravitational wave (or graviton) and the vector electromagnetic wave (or photon) have zero mass and charge four-currents, respectively. These waves can not generate themselves gravitational and electromagnetic fields, and our gravitons, like photons, do not couple to each other.

The numerical results (9) and (10) are well known and were proposed by many authors, but we have derived these results for light in the flat world space. Accepting the verified agreement of $\Delta\varphi$ from (7), Δt from (9), and φ_∞ from (10) with the relevant measurements, one may conclude that the Mercury perihelion precession, the radar echo delay, and the gravitational light deflection by the Sun reinforce the credibility of the flat Universe.

4. Proper four-space for the charged mass

The observable evolution of matter is three-dimensional in spite of the fact that the proper four-space or other high dimensional manifolds can be employed for a self-consistent description of any selected mass. Geometry of proper high dimensional manifolds may differ from Euclidean geometry of their 3D intersection called the world space. This provides an opportunity to implicate electric charges into the metric tensor of the formally introduced pseudo-Riemannian four-space $x = x_N$.

In order to describe one selected mass m_N with the electric charge q_N we employed the symmetrical involvement of masses and charges into the proper particle four-momentum (2) or the total four-velocity $V_{N\mu} \equiv m_N^{-1} P_{N\mu}$,

$$\begin{aligned} V_{N\mu}(x) &= \mathcal{V}_\mu + \sum_K^{all} [-Gm_K a_{K\mu}^-(x) + m_N^{-1} q_N q_K a_{K\mu}^+(x)] + \partial_\mu \phi_N(x) \\ &\equiv \mathcal{V}_\mu + U_\mu(x) = \left\{ \frac{1}{\sqrt{1-v^2}}; -\frac{\delta_{ij}v^j}{\sqrt{1-v^2}} \right\} + \left\{ \frac{(\sqrt{g_{oo}}-1)}{\sqrt{1-v^2}}; -\frac{g_i\sqrt{g_{oo}}}{\sqrt{1-v^2}} \right\}, \quad (14) \end{aligned}$$

where $U_\mu(x) \equiv \sum_K^{all} [-Gm_K a_{K\mu}^-(x) + m_N^{-1} q_N q_K a_{K\mu}^+(x)] + \partial_\mu \phi_N \equiv B_\mu^{\neq N}(x) + S_\mu^N(x)$, $S_\mu^N(x) \equiv -Gm_N a_{N\mu}^-(x) + \partial_\mu \phi_N(x) + m_N^{-1} q_N \sum_K^{all} q_K a_{K\mu}^+(x)$, $\mathcal{V}_\mu(x) \equiv \delta_\mu^\alpha V_\alpha = \{\beta^{-1}, -\beta^{-1}\delta_{ij}v^j\}$, and $\beta = |ds|/d\tau = |ds|/(ds^2 + dl^2)^{-1/2} = \sqrt{1 - \delta_{ij}v^i v^j}$.

By employing the coupled pre-potentials $a_{K\mu}^\pm$ for the electromagnetic, $q_K a_{K\mu}^\pm$, and gravitational, $-Gm_K a_{K\mu}^-$, field potentials, one can introduce for every charged mass the proper four-dimensional space x_N^μ with affine connections generated by both external masses and electric charges. The total (canonical) four-momentum of the charged mass in its proper pseudo-Riemannian four-space takes the "old", mechanical view, $P_N^\mu = m_N dx_N^\mu/ds_N$, $P_{N\nu} = m_N g_{\mu\nu}^N dx_N^\mu/ds_N = m_N (\delta_\nu^\alpha V_\alpha + B_\nu) + q_N A_\nu + m_N \partial_\mu \phi_N$, $ds_N^2 = g_{\mu\nu}^N dx_N^\mu dx_N^\nu$, $P_{N\mu} P_N^\mu = m_N^2$. In principle, all charges q_K and masses m_K may contribute into the proper (canonical) metric tensor for the mass m_N ,

$$\begin{cases} g_{oo}^N = (1 + \beta_N U_o)^2 \\ g_{oi}^N = (1 + \beta_N U_o) \beta_N U_i \\ g_{ij}^N = \beta_N^2 U_i U_j - \delta_{ij} \\ g_{oi}^N g_{oj}^N (g_{oo}^N)^{-1} - g_{ij}^N = \delta_{ij} \end{cases}, \quad (15)$$

into the proper (canonical) tetrad ($g_{\mu\nu}^N \equiv g_{\mu\nu} = \eta_{\alpha\beta} e_{N\mu}^\alpha e_{N\nu}^\beta$),

$$e_{N\mu}^\alpha(x) = \delta_\mu^\alpha + \delta^{\alpha o} \beta_N U_\mu(x) \equiv \delta_\mu^\alpha + \delta^{\alpha o} \beta_N (B_\mu^{\neq N} + S_{N\mu}) \equiv \delta_\mu^\alpha + \delta^{\alpha o} \beta_N (V_\mu - \mathcal{V}_\mu), \quad (16)$$

and into the affine connection in the proper (canonical) four-space x_N ,

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2}g_N^{\lambda\rho}(\partial_\mu g_{\nu\rho}^N + \partial_\nu g_{\mu\rho}^N - \partial_\rho g_{\mu\nu}^N). \quad (17)$$

The point is that all potentials in (15)-(17) are not observable notions and the gauge field ought to be selected properly in order to avoid nonphysical results. One can verify from (15) that the three-space metric tensor $\gamma_{ij} \equiv g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} = \delta_{ij}$ corresponds to Euclidean 3D sub-space under any gauge transformations. These six bounds admit only four independent functions, $\beta_N U_\mu$, for the ten different metric components in (15). And these four functions have three transitional degrees of freedom due to the particle velocity v_N^i in β_N , four gravitational degrees of freedom due to the external field potential $B_\mu^{\neq N}$, and three rotational degrees of freedom due to the internal self-potential S_μ^N , with $S_\mu^N P_\mu^N = 0$. We prove below that the proper particle time $d\tau_N$ is not related to new degrees of freedom, *i.e.* there are only ten degrees of freedom in the pseudo-Riemannian metric tensor (15) for the most general case.

The physical three-velocity of the mass N in the flat world space, $dx_N^i/d\tau_N \equiv v_N^i \equiv v^i = \delta^{ij}v_j$, is determined by the proper time $d\tau_N$. The proper time of one selected charged particle N, $d\tau_N \equiv \beta^{-1}|ds| = |\sqrt{g_{oo}}(dx^o - g_i dx^i)| = |(1 + \beta U_o)dx^o + \beta U_i dx^i| = |dx^o + \beta U_\mu dx^\mu| = |dx^o + \beta^2 U_\mu V^\mu d\tau \text{sign } ds|$ and $\text{sign } ds = \text{sign } dx^o$, depends on the external gravitational potential $B_\mu^{\neq N}$,

$$\begin{aligned} \frac{d\tau_N}{|dx^o|} &\equiv \frac{1}{1 - \beta_N^2(\tau_N)U_\mu V_N^\mu(\tau_N)} \equiv \frac{1 + \beta_N(\tau_N)U_o}{1 - \beta_N(\tau_N)U_i v_N^i(\tau_N)} \\ &= \frac{1}{1 - \beta_N^2(\tau_N)B_\mu^{\neq N} V_N^\mu(\tau_N)} \equiv \frac{1 + \beta_N(\tau_N)B_o^{\neq N}}{1 - \beta_N(\tau_N)B_i^{\neq N} v_N^i(\tau_N)}. \end{aligned} \quad (18)$$

We used in the equation (18) the physical, spin-type gauge with $S_\mu^N V_N^\mu = 0$, when $S_o(1 - \beta U_i v^i) + S_i v^i(1 + \beta U_o) = S_o(1 - \beta B_i^{\neq N} v^i) + S_i v^i(1 + \beta B_o^{\neq N}) = 0$. Then the self-potential $S_\mu \equiv S_\mu^N$ or the particle mass m_N and charge q_N do not contribute to the proper particle time $d\tau_N$ in the physical velocity $v_N^i = dx_N^i/d\tau_N$. The proper four-interval $ds_N^2 = d\tau_N^2 - \delta_{ij}dx_N^i dx_N^j$ of the electrically charged particle N depends only on the external gravitational potential $B_\mu^{\neq N}(x)$ and takes pseudo-Euclidean form in the absence of external masses.

By taking into account that $\beta_N \equiv \beta = |ds|/(ds^2 + dl^2)^{1/2}$, one can read (18) as follows,

$$\frac{d\tau_N}{dt} \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + U_\mu V_N^\mu(\tau_N) \frac{ds_N^2}{dt^2}} = \frac{1}{2} + \sqrt{\frac{1}{4} + B_\mu^{\neq N} V_N^\mu(\tau_N) \frac{ds_N^2}{dt^2}}. \quad (19)$$

The basic interval relation $ds_N^2 + dl_N^2 \equiv (dx_N^o + \beta U_\mu dx^\mu)^2 \equiv (dx_N^o - \alpha_N ds_N)^2$, with $\alpha_N \equiv -\beta U_\mu V^\mu = -\beta B_\mu^{\neq N} V^\mu \equiv -(B_o^{\neq N} + B_i^{\neq N} v^i)/(1 + \beta B_o^{\neq N})$, $ds_N^2 \equiv g_{\mu\nu}^N dx_N^\mu dx_N^\nu$, $dl_N^2 \equiv \delta_{ij} dx_N^i dx_N^j$, and $dt_N^2 \equiv \delta_{oo} dx_N^o dx_N^o$, may be also represented in the following form,

$$ds^2 \equiv \left(\frac{\sqrt{(dx^o)^2 - dl^2(1 - \alpha_N^2)} \mp \alpha_N dx^o}{(1 - \alpha_N^2)} \right)^2 \approx \frac{dt^2}{(1 + \alpha_N)^2} - \frac{dl^2}{(1 + \alpha_N)}, \quad (20)$$

when $(1 - \alpha_N^2)dl^2/dt^2 \ll 1$. By selecting the only one sign in (20), we used from (18) that $ds^2 = dt^2(1 + \alpha_N)^{-2}$ when $dl^2 = 0$. Notice that $U_\mu V^\mu = B_\mu^{\neq N} V^\mu < 0$

and $\alpha_N > 0$ for strong gravitational fields, *i.e.* there is no Schwarzschild-type peculiarity or the black hole in the nonlinear interval equations (18)-(20). But all these nonlinear equations lead to Schwarzschild's time dilation $d\tau/dt \approx 1 - GMr^{-1}$ [2] under weak gravitational fields $GMr^{-1} \ll 1$ and non-relativistic velocities $(1 - \beta) \ll 1$.

The following summary of main relations between the proper metric tensor, $g_{\mu\nu} \equiv g_{\mu\nu}^N(x)$, the proper four-velocity, $V_\mu \equiv m_N^{-1}P_{N\mu}(x)$, and the field potential, $U_\mu \equiv U_{N\mu}(x) \equiv B_\mu^{\neq N}(x) + S_\mu^N(x)$, may be useful for our references,

$$\begin{aligned}
g_{oo} &= (1 + \beta U_o)^2, \quad g_{oi} = (1 + \beta U_o)\beta U_i, \quad g_{ij} = \beta^2 U_i U_j - \delta_{ij}, \\
g^i &= -g^{oi} = \gamma^{ij} g_j = g_i = -g_{oi} g_{oo}^{-1} = -\beta U_i (1 + \beta U_o)^{-1}, \\
g^{oo} &= g_{oo}^{-1} - g_i g^i = (1 - \beta^2 U_i U_j \delta^{ij})(1 + \beta U_o)^{-2}, \quad \gamma_{ij} = \gamma^{ij} = -g^{ij} = \delta_{ij}, \\
V_\mu &= \{\beta^{-1} + U_o; -\beta^{-1} v_i + U_i\} = \mathcal{V}_\mu + U_\mu = g_{\mu\nu} V^\nu, \\
V^\mu &= \{(\beta^{-1} + U_o); V^i\} = \{\beta^{-1} - (U_o + U_i v^i)(1 + \beta U_o)^{-1}; \beta^{-1} v^i\}, \\
V_\mu V^\mu &= g_{oo}(V^o - g_i V^i)^2 - \delta_{ij} V^i V^j = V_o^2 g_{oo}^{-1} - \beta^{-2} v^2 = 1; \\
\text{Gauge: } S^o &\equiv [S_o(1 - \beta B_i^{\neq N} v^i)(1 + \beta B_o^{\neq N})^{-1} + S_i v^i](1 + \beta U_o)^{-1} = 0, \\
\text{with } U^o &\equiv B_o^o + S^o = B_{\neq N}^o = (B_o^{\neq N} + \beta B_i^{\neq N} v^i)(1 + \beta B_o^{\neq N})^{-1}, \\
S^i &\equiv 0, \quad (1 - \beta U_i v^i)(1 + \beta U_o)^{-1} = (1 - \beta B_i^{\neq N} v^i)(1 + \beta B_o^{\neq N})^{-1}, \\
V^\mu &= \{\beta^{-1} - (B_o^{\neq N} + B_i^{\neq N} v^i)(1 + \beta B_o^{\neq N})^{-1}; \beta^{-1} v^i\}, \\
S_\mu V^\mu &= 0, \quad S_\mu S^\mu = 0, \quad V_\mu V^\mu = (\mathcal{V}_\mu + B_\mu^{\neq N} + S_\mu) V^\mu = 1. \quad (21)
\end{aligned}$$

It is interesting that the contravariant component $P_N^i \equiv m_N V^i = m_N \beta^{-1} v^i$ does not depend on external potentials (because $U^i \equiv 0$), while the covariant three-momentum $P_{Ni} = -m_N \beta^{-1} v_i + m_N U_i$ depends on them. This means that the electromagnetic potentials A_μ and A^μ , as well as the total potentials U_μ and U^μ , are not themselves four-vectors.

The metric tensors $g_N^{\mu\nu}$ can not be applied for rising indexes of any one summand in $m_N V_\mu = m_N \mathcal{V}_\mu + m_N B_\mu + m_N \partial_\mu \phi_N + q_N A_\mu$, despite $g_N^{\mu\nu} V_\mu = V^\nu$ for the total (canonical) four-velocity $V_\mu \equiv V_{N\mu}$ of the charged mass N. The physical gauge $S_\mu P^\mu = 0$ in $V_\mu P^\mu = (\mathcal{V}_\mu + B_\mu^{\neq N} + S_\mu) P^\mu = m_N$ means that the self-potential (and the charge q_N) does not directly contribute to the particle rest mass m_N .

There is no metrics to relate the self-potentials $S_\mu \neq 0$ and $S^\mu = 0$, which are associated, in particular, with the electromagnetic potentials $q_N A_\mu$ and $q_N A^\mu$. A massless electric charge, $q_N \neq 0$ with $m_N = 0$, can not exist in practice because the electromagnetic part, $q_N A_\mu(x)$, of the canonical four-momentum does not maintain a definite tensor nature in the proper four-space x_N^μ .

5. Point macroscopic and extended microscopic particle states

5.1. The macroscopic geodesic motion of the point charge

The coupled gravitational and electromagnetic potentials in (2) or (14) can lead in principle to balanced vector forces under the free motion of the charged particle. That is why we may generalize the equivalence principle on the proper canonical four-space x_N^μ with the Christoffel connections (17). A macroscopic

geodesic equation may be derived due to the covariant conservation of the localized particle four-momentum $P_{N\mu}(\xi_N)$ at every path point ξ_N in the curved canonical four-space, where $V_{N\mu}V_N^\mu \equiv 1$, $V_N^\nu \nabla_\mu^N V_{N\nu} \equiv 0$, and $\Gamma_{\rho\lambda}^\mu = \Gamma_{\lambda\rho}^\mu$,

$$\begin{aligned} \frac{DP_{N\mu}(\xi_N)}{ds_N(\xi_N)} &= V_N^\nu (\nabla_\nu P_{N\mu} - \nabla_\mu^N P_{N\nu})_{x \rightarrow \xi_N} = P_N^\nu (\partial_\nu V_\mu - \partial_\mu V_\nu) \\ &= P_N^\nu (\partial_\nu \mathcal{V}_\mu - \partial_\mu \mathcal{V}_\nu) - P_N^\nu (\partial_\mu B_\nu - \partial_\nu B_\mu) - q_N V_N^\nu (\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= m_N V^\nu (\partial_\nu \mathcal{V}_\mu - \partial_\mu \mathcal{V}_\nu)_{x \rightarrow \xi_N} - m_N V^\nu \sum_K^{K \neq N} (-Gm_K) f_{\mu\nu}^{K-}(\xi_N) \\ &\quad - q_N V^\nu \sum_K^{K \neq N} q_K f_{\mu\nu}^{K+}(\xi_N) - (q_N^2 - Gm_N^2) V^\nu f_{\mu\nu}^N(x)_{x \rightarrow \xi_N} = 0. \end{aligned} \quad (22)$$

The retarded Lorentz force for the electric charge, $q_N V_N^\nu (\partial_\mu A_\nu^{\neq N} - \partial_\nu A_\mu^{\neq N}) = q_N V_N^\nu \sum_K^{K \neq N} q_K (\partial_\mu a_\nu^{K+} - \partial_\nu a_\mu^{K+}) = q_N V_N^\nu \sum_K^{K \neq N} q_K f_{\mu\nu}^{K+}(\xi_N)$, is accompanied in (22) by the advanced gravitational force for the mass with the similar vector nature, $m_N V_N^\nu (\partial_\mu B_\nu^{\neq N} - \partial_\nu B_\mu^{\neq N}) = m_N V_N^\nu \sum_K^{K \neq N} (-Gm_K) (\partial_\mu a_\nu^{K-} - \partial_\nu a_\mu^{K-}) = m_N V_N^\nu \sum_K^{K \neq N} (-Gm_K) f_{\mu\nu}^{K-}(\xi_N)$. The self-force $(q_N^2 - Gm_N^2) V^\nu f_{\mu\nu}^N(x)_{x \rightarrow \xi_N}$ originates from the net energy loss under generation of electromagnetic and gravitational waves. Below this self-force will be bound with the integral changes of the self-field tensor density $\theta_\nu^\mu(x)$ of the microscopic particle states over all vacuum points $x \neq \xi_N$. Notice for (22) that the gauge term is absent due to a tensor equality $P^\nu (\partial_\nu \partial_\mu \phi_N - \partial_\mu \partial_\nu \phi_N) = 0$ and there are only three independent equations due to scalar equalities $P^\mu DP_\mu \equiv 0$ and $P^\mu P^\nu (\partial_\nu V_\mu - \partial_\mu V_\nu) \equiv 0$.

The macroscopic geodesic equations (22), based on the canonical four-space with metric relations (15)-(17) and the physical gauge $S^o = S^\mu = 0$, $(\mathcal{V}_\mu + B_\mu^{\neq N})V^\mu = 1$, corresponds to the classical Minkowski-Lorentz analogue $Du_\mu/ds = m_N^{-1} q_N u^\nu (\partial_\mu A_\nu - \partial_\nu A_\mu)$, based on the four-space concept with pure mechanical connections and $u_\mu u^\mu = 1$.

One can read the general geodesic equation (22) in the non-relativistic limit, where $q_N (E + v \times B)_i + m_N (E_g + v \times B_g)_i = m_N [V^o (\partial_i \mathcal{V}_o - \partial_o \mathcal{V}_i) + V^j (\partial_i \mathcal{V}_j - \partial_j \mathcal{V}_i)] \approx m_N [\partial_o v_i + (v^j \nabla_j) v_i] = m_N dv_i/dt$ and $v^i (m_N E_{gi} + q_N E_i) = m_N v^i dv_i/dt$. A self-action here may be found from a non-divergence part of the proper four-potential [8], $a_{N\mu}(x)_{x \rightarrow \xi_N} \approx \{0, -2\dot{v}/3\}$, which lead to $E_i^{self} = q_N 2\ddot{v}/3$, $E_{gi}^{self} = -Gm_N 2\ddot{v}/3$, $B_i = B_{gi} = 0$, and $m_N \dot{v}_i = -2(Gm_N^2 - q_N^2) \ddot{v}_i/3$.

The physical gauge restriction, $S_\mu^N(x) P_N^\mu(x) = 0$, and the covariant conservation $DP_N^\mu/ds_N = 0$, lead to a spin-type relation for the self-momentum $m_N S_{N\mu}$,

$$P_N^\mu \frac{DS_{N\mu}}{ds_N} = m_N V^\mu \left(\frac{dS_{N\mu}}{ds_N} - \Gamma_{\mu\nu}^\rho V^\nu S_{N\rho} \right). \quad (23)$$

This equation reads $V^o V^o \Gamma_{oo}^i S_i = 0$ in the particle rest frame, where $V^i = 0$ and $S_o = 0$. From here one finds a scalar conservation $\partial_o (m_N^2 S_i S_j \delta^{ij}) = 0$ with the proper self-momentum $m_N S_i(\xi_N) = m_N \partial_i \phi_N(\xi_N) + (q_N^2 - Gm_N^2) a_{Ni}(\xi_N) \neq 0$, while $m_N S_o(\xi_N) = m_N \partial_o \phi_N(\xi_N) - (Gm_N^2 - q_N^2) a_{No}(\xi_N) = 0$, in the absence of external gravitational and electromagnetic fields. The self-potential $S_{N\mu}$ has only three degrees of freedom and it is not a four-vector. This potential may be associated with an internal angular momentum or spin of the elementary object N with the net rest mass m_N .

5.2. Maxwell-type vector equations for the particle mass and charge

The macroscopic geodesic motion of the point source-outlet N along the classical path ξ_N depends on elementary electromagnetic, $q_N f_{\mu\nu}^{K+}(\xi_N)$, and gravitational, $-Gm_K f_{\mu\nu}^{K-}(\xi_N)$, fields related by zero four-intervals with distant sources-outlets K at point vertices ξ_K . The equation (22) is not complete if we do not know how to relate these skew-tensor fields with their moving sources-outlets.

One way to relate the gravitational field to the mass corresponds to the tensor Einstein equation [1], which may be derived, for example, after the Hilbert variation [17] of a system action with respect to ten variations $\delta g_{\mu\nu}$ ($g_{\mu\nu} V^\mu V^\nu \neq 1$ before the variations). Recall that there is no one ten-component field in (15) behind the symmetric tensor $g_{\mu\nu}^N$ and its ten degrees of freedom have different physical origin. These degrees of freedom may be also traced in the four-vector variable $P_{N\mu}(x)$. We have first to define an elementary action for a particle N and its self field in order to select dynamical variables and derive Lagrange variational equations.

At first we accept an introduction of the particle in terms of extended microscopic states over all vacuum space by keeping the localized or point particle state exclusively for space-time averaged, macroscopic relations. The wave-particle dualism, propounded in 1905 by Einstein in his quantum theory of light, was fruitfully inverted by de Broglie in 1923, when he first departed from the point particle notion and developed the field analogy of the particle with the light wave [18]. By following this verified way, we may postulate that the particle is not localized under its microscopic states no matter the classical or the wave approach is taken into consideration. In other words, we are going to reinvest the Einstein - de Broglie particle-wave dualism from Wave Mechanics back to Classical Electrodynamics and Gravitation through a postulated concept of the dual particle states (point macroscopic and nonlocal microscopic ones). The observed localization of the classical particle in our approach will take place only for the averaged, macroscopic state.

The particle integration into the classical field structure had been anticipated by Einstein at the end of his life: "We could regard matter as being made up of regions of space in which the field is extremely intense... There would be no room in this new physics for both field and matter, for the field would be the only reality" (translation [19]). His analysis of the Maxwell-Lorentz equations revealed the following: "The combination of the idea of a continuous fields with that of discontinuous material points in space strikes me as contradictory. A coherent field theory would consist exclusively of continuous elements not only in time but in every point of space. Hence the material particle cannot possible be a fundamental concept in any field theory. Thus, quite apart from the fact that it ignores gravitation, Maxwell's theory cannot be considered as a complete theory" (translation [20]). The Lorentz introduction of point field sources into microscopic equations is "an attempt which we have called intellectually unsatisfying" [20], and dissatisfied Einstein criticised his gravitational equation with the point sources: "it resembles a building with one wing build of resplendent marble, and the other build of cheap wood". The problem of point particles in the classical field equations has not been resolved satisfactorily yet that may be considered as a motivation for our introduction of the extended microscopic states along with the averaged macroscopic state for the same entity, called the particle.

The Dirac delta-operator, with $\int dx^4 P_{N\mu}(x) \delta^3(\mathbf{x} - \xi_N[p]) \delta(x^0 - \xi_N^0[p]) = P_{N\mu}(\xi_N[p])$, will assist us to interpret the particle in terms of the extended microscopic states (or continuous virtual fluctuations) with the local particle

four-momentum $P_{N\mu}(x) = m_N V_\mu(x)$ and the local four-velocity $V^\mu(x)$ at all vacuum points $x^\mu \neq \xi_N^\mu$. We will consider the particle N in microscopic relations as a nonlocal continuous fraction of an elementary object N, where the relevant particle densities are locally bound with induced self fields $q_N f_{\mu\nu}^N(x)_{x \neq \xi_N}$ and $-Gm_N f_{\mu\nu}^N(x)_{x \neq \xi_N}$. Such interpretation of the particle for microscopic states can be agreed both with macroscopic particle peculiarities and with an introduction of the material vacuum through a global intersection of elementary particle-field objects with zero elementary mass/charge/energy densities. Again, the particle entity without its self-fields is not a complete elementary object in our consideration.

Now we can define the tensor density $W_{\mu\nu}^N(x) \equiv \nabla_\mu^N P_{N\nu}(x) - \nabla_\nu^N P_{N\mu}(x) = \partial_\mu P_{N\nu}(x) - \partial_\nu P_{N\mu}(x)$ of the extended particle states and the self-field tensor $f_{\mu\nu}^N(x) \equiv \nabla_\mu^N a_{N\nu}(x) - \nabla_\nu^N a_{N\mu}(x) = \partial_\mu a_{N\nu}(x) - \partial_\nu a_{N\mu}(x)$ at the same points of the proper four-space $x_N^\mu = x^\mu$ (for brief). A local coupling of the microscopic particle states and their fields takes place overall x_N^μ and a scalar contraction of their tensor densities should be put into the microscopic action \mathcal{S}_N^{mic} of the complete elementary particle-field object N,

$$\begin{aligned} \mathcal{S}_N^{mic} = & - \int d^4x \sqrt{-g_N} \int d\xi_N^o[p] \frac{\delta^3(\mathbf{x} - \xi_N[p]) \delta(x^o - \xi_N^o[p]) P_{N\mu}^{x \neq \xi_K}(x)}{\sqrt{-g_N(x)}} \frac{dx^\mu}{d\xi_N^o[p]} \\ & - \int_{x \neq \xi_K, \xi_N} \frac{d^4x \sqrt{-g_N(x)}}{16\pi} \left[g_N^{\mu\rho}(x) g_N^{\nu\lambda}(x) W_{\mu\nu}^N(x) f_{\rho\lambda}^N(x) + g_{\mu\nu}^N(x) G^{-1} r_N^{\mu\nu} \right] \\ = & - \int d^4x \sqrt{-g_N} \left(P_{N\mu}^{x \neq \xi_K} \frac{\delta^3(\mathbf{x} - \xi_N[x^o]) dx^\mu}{\sqrt{-g_N(x)} dx^o} + \frac{W_{N\mu\nu}^{x \neq \xi_K} f_{x \neq \xi_N}^{N\mu\nu}}{16\pi} + \frac{r_N(x)}{16\pi G} \right), \quad (24) \end{aligned}$$

where $\sqrt{-g_N(x)} \equiv \sqrt{\gamma_N g_{oo}^N(x)} \equiv \sqrt{g_{oo}^N(x)}$, $r_N(x) \equiv g_{\mu\nu}^N(x) r_N^{\mu\nu}(x)_{x \neq \xi_N, \xi_K}$ is the scalar curvature of the four-space x_N^μ at peculiarity free points, $f_{N\mu\nu}(x)$ is not define at ξ_N , and $P_{N\mu}(x)$ is not define at all ξ_K , when $\xi_K \neq \xi_N$.

By applying the delta-operator in (24), one may find an action $\mathcal{S}_N^{mac} = - \int dp P_{N\mu}(\xi_N[p]) (d\xi_N^\mu[p]/dp)$ for the macroscopic, localized particle at its classical path ξ_N , where there are no self-fields in our definition. The path variations $\delta\xi_N[p]$ at these bare particle points in \mathcal{S}_N^{mac} lead to $m_N V_N^\nu(\xi_N) W_{\mu\nu}^N(\xi_N) = 0$, *i.e.* to the macroscopic geodesic equation (22) under its first integral $V_{N\mu}(\xi_N) V_N^\mu(\xi_N) = 1$. The geodesic equation for the extended microscopic states at the field points $x \neq \xi_N$ will be derived below after the action variations with respect to δx_N^μ .

The variations of (24) with respect to $\delta P_{N\mu}(x)$ at pure field points $x \neq \xi_N, \xi_K$ correspond to a microscopic Lagrange equation

$$I_N^\nu(x)_{x \neq \xi_N, \xi_K} \equiv i_N^\nu(x) - \frac{\nabla_\mu^N f_N^{\mu\nu}(x)}{4\pi} \equiv \frac{\delta^3(\mathbf{x} - \xi_N[x^o]) dx^\mu}{\sqrt{-g_N(x)} dx^o} - \frac{\nabla_\mu^N f_N^{\mu\nu}(x)}{4\pi} = 0, \quad (25)$$

where $f_N^{\mu\nu}(x) \equiv g_N^{\mu\rho}(x) g_N^{\nu\lambda}(x) f_{\rho\lambda}^N(x)_{x \neq \xi_N, \xi_K}$ and $\sqrt{-g_N(x)} \equiv \sqrt{g_{oo}^N(x)}$. Notice that not all components of the skew-symmetric tensors are independent under variations [19]: the relations $\delta W_{\mu\nu}^N(x) = -\delta W_{\nu\mu}^N(x)$ were taken into account. The variable $P_{N\mu}(x)$ has four degrees of freedom before the variations because $P_{N\mu} dx^\mu/dp \neq const$ in the action (24). We may define $dp = ds_N$ for the path parameter p after the variations and then use $V_{N\mu}(x) V_N^\mu(x) = 1$ in the equations of motion.

At first glance the Maxwell-type variational equation (25) seems not new at all. The only point it is not specified neither for the mass, nor for the electric or other kind of charge. This basic equation for the microscopic particle states and their fields manifests in general that any kind of the particle matter four-flow $i_N^\nu(x)$ or pre-current is to be locally screened by an induced appropriate pre-field $f_N^{\mu\nu}(x)$ at all peculiarity free points, called vacuum or field points in macroscopic physics. Indeed, different solutions of this microscopic equation may be associated with differently charged particle-field matter. Below we apply (25) only to the mass and to the electric charge in order to derive macroscopic equations and to relate the macroscopic gravitational fields to the advanced Lienard-Wiechert potentials, while the macroscopic electromagnetic fields with the retarded ones. But first we rewrite (25) for the relevant microscopic equations with the active gravitational mass m_N and the electric charge q_N , respectively,

$$\begin{cases} -4\pi G\sqrt{g_{oo}^N}m_N i_N^\nu(x)_{x \neq \xi_N, \xi_K} = \partial_\mu \sqrt{g_{oo}^N}[-Gm_N f_N^{\mu\nu}(x)]_{x \neq \xi_N, \xi_K} \\ 4\pi\sqrt{g_{oo}^N}q_N i_N^\nu(x)_{x \neq \xi_N, \xi_K} = \partial_\mu \sqrt{g_{oo}^N}q_N f_N^{\mu\nu}(x)_{x \neq \xi_N, \xi_K} \end{cases} \quad (26)$$

Here there is only a local coupling of the fields with the microscopic currents $\{-Gm_N/q_N\}i_N^\mu(x)_{x \neq \xi} \equiv \{-Gm_N/q_N\}\delta^3(\mathbf{x} - \xi_N[p])dx^\mu/\sqrt{-g_N}dx^o$ of the extended particle in $x^\mu \neq \xi_N^\mu$. The microscopic particle currents (or virtual particle fluctuations) reshaping microscopic fields overall space with an infinite speed $C = \infty$, otherwise another Dirac operator, $\hat{\delta}^4(x - \xi_N[p]) = \delta^3(\mathbf{x} - \xi_N[p])\delta(x^o - \xi_N^o[p] \pm C^{-1}|\mathbf{x} - \xi_N[p]|)$, is to be used in the action (24). The nonlocal nature of the particle on the microscopic level is responsible for instantaneous updating of both outgoing and incoming microscopic fields that is not in disagreement with the Special Relativity restrictions for macroscopic fields.

The advanced/retarded relations of macroscopic self-potentials with their mass/charge at ξ_N do arise from (26), but only for the macroscopic states averaged over all microscopic states (or virtual fluctuations for macro-world) under "small macroscopic - large microscopic" time scales $\Delta x^o \sim t^o$. At these scales the particle and its fields disintegrate in space, and the macroscopic generalization of (25) takes the following form,

$$\begin{cases} i_{Nmac}^\mu(x)_{x \neq \xi_N} = 0, & f_{Nmac}^{\mu\nu}(x)_{x \neq \xi_N} \neq 0, & \nabla_\mu^N f_{Nmac}^{\mu\nu}(x)_{x \neq \xi_N} = 0, \\ i_{Nmac}^\mu(\xi_N) \neq 0, & f_{Nmac}^{\mu\nu}(\xi_N) = 0, & \nabla_\mu^N f_{Nmac}^{\mu\nu}(x)_{x = \xi_N} = 0. \end{cases} \quad (25a)$$

The macroscopic disintegration of masses or charges with their advanced or retarded, respectively, fields in (25a) should be considered as a fundamental property of this kind of matter. In principle, one may find some more field solutions of the general equation (25), which are not related to space disintegration with extended charges. Such charges cannot be observed in macro-world and they are out of plans to consider macroscopic gravity and electrodynamics.

One may say that the microscopic particle states or virtual particle fluctuations are screened completely by locally induced charged fields, and every elementary material object has neither the net mass four-current density, $m_N I_N^\nu(x)_{x \neq \xi} = 0$, nor the electric four-current density, $q_N I_N^\nu(x)_{x \neq \xi} = 0$, at all field points. There are no screening self-fields in the vertex ξ_N , where the elementary object is represented by only the particle fraction. Thus the particle exhibits in ξ_N its finite mass/charge four-current on behalf of the complete elementary object, which becomes available for observations at this point.

Another microscopic equations and their macroscopic generalizations for the incoming and outgoing field densities may be derived directly from their skew-

symmetrical tensor structures $f_{\mu\nu}^N(x)_{x \neq \xi_N}$,

$$\begin{cases} (-Gm_N)[\partial_\mu f_{N\nu\delta}(x) + \partial_\nu f_{N\delta\mu}(x) + \partial_\delta f_{N\mu\nu}(x)]_{x \neq \xi_N}^{mic/mac} = 0 \\ q_N[\partial_\mu f_{N\nu\delta}(x) + \partial_\nu f_{N\delta\mu}(x) + \partial_\delta f_{N\mu\nu}(x)]_{x \neq \xi_N}^{mic/mac} = 0 \end{cases} \quad (27)$$

Notice that the macroscopic equations (25a) and (27) admit retarded wave solutions for the field $q_N f_{Nw}^{\mu\nu}(x)_{x \neq \xi_N, \xi_K}$ with a zero electric current, $q_N \nabla_\mu f_{Nw}^{\mu\nu} \equiv q_N g_{oo}^{-1/2} \partial_\mu \sqrt{g_{oo}} f_{Nw}^{\mu\nu} = 0$, and advanced wave solutions $[-Gm_N f_{Nw}^{\mu\nu}(x)]_{x \neq \xi_N, \xi_K}$ with a zero mass current, $(-Gm_N) \nabla_\mu f_{Nw}^{\mu\nu} \equiv -Gm_N g_{oo}^{-1/2} \partial_\mu \sqrt{g_{oo}} f_{Nw}^{\mu\nu} = 0$. They are anti-waves to each other because the spherical electromagnetic wave is moving from its point source ξ_N , while the spherical gravitational wave is moving with the same light speed c toward its point outlet ξ_N .

Only the particle four-currents may induce the interaction fields based on the uncharged pre-field $f_{\mu\nu}$. These fields affect another particles and couple material entities with nonzero mass/charge four-currents. The gravitational/electromagnetic mass/charge dimensional waves from (26)-(27) have formally the same mass / charge density, respectively, as their parent particle. But these "massive" and "charged" fields with zero mass and charge four-currents do not induce any interaction fields. In other words, the gravitons and the photons have only zero active gravitational masses (as well as zero passive and inertial masses, and zero passive and active electric charges) and they do not couple to each other in our consideration.

It is important for macroscopic causality that the advanced gravitational wave or their packet would come from infinity to the point vertex-outlet only through infinite time intervals and, therefore, this gravitational wave never be registered in practice or cross the retarded electromagnetic wave. From the other side, the instantaneous generation of gravitational waves in infinity takes energy from the accelerated mass and lead, for example, to the radiation damping in oscillation of binary pulsars. In general the mass exhibits inertia or resists to any acceleration by generating the advanced waves in infinity, while the non-inertial charge welcome self-accelerations by generating the retarded waves.

We should recall from [5], for example, for the uniformly moving particle that its Newton and Coulomb forces, derived from the advanced and retarded Lienard-Wiechert potentials, respectively, are both directed toward the instantaneous particle position, rather than toward different (advanced and retarded) positions. These forces are always parallel due to the Lorentz invariance in all inertial frames of references, including the particle rest frame, and forces cannot be promptly used for verifying the advanced nature of macroscopic gravitation. Aberrations of Sun's electromagnetic and gravitational waves would have different signs, could causality be violated and the gravitation wave be registered one day.

5.3. Superfluid behaviour of microscopic particle states

Now we derive one more Lagrangian equation by varying the global ensemble action $\sum_N^\infty \mathcal{S}_N^{mic}$ with respect to $\delta a_{N\mu}$,

$$\nabla_\mu W_N^{\mu\nu} = 0, \quad (28)$$

where we used $\delta P_{K\mu}(a_{N\mu}) = const \delta a_{N\mu}$ and (25).

We also may write directly from the definition $W_{\mu\nu}^N(x) \equiv \nabla_\mu^N P_{N\nu}(x) - \nabla_\nu^N P_{N\mu}(x)$ another relation for the microscopic particle states in all field points $x \neq \xi_N, \xi_K$ or in vacuum,

$$\partial_\mu W_{\nu\delta}^N(x) + \partial_\nu W_{\delta\mu}^N(x) + \partial_\delta W_{\mu\nu}^N(x) = 0 \quad (29)$$

with $m_N [\partial_\mu M_{\nu\delta}^N(x) + \partial_\nu M_{\delta\mu}^N(x) + \partial_\delta M_{\mu\nu}^N(x)] = 0$ when $q_N = 0$.

One could formally decompose the total particle four-momentum $P_\mu^N(x)$ in (2) into a gravitomechanical part and an electrical one. Then the tensor density $W_{\mu\nu}^N(x)$ of extended particle states in vacuum would be formally divided into a gamvitomechanical part and an electric part (with q_N), $W_{N\mu\nu}(x) = m_N M_{N\mu\nu}(x) + q_N F_{\mu\nu}(x)$, where $M_{\mu\nu}(x) \equiv w_{\mu\nu}(x) + G_{\mu\nu}(x)$, $w_{\mu\nu}(x) \equiv \partial_\mu \mathcal{V}_\nu(x) - \partial_\nu \mathcal{V}_\mu(x)$, $\mathcal{V}_\mu(x) \equiv \{\beta^{-1}, -\beta^{-1}v_i\}$, $\beta = \sqrt{1 - v_i v^i}$, $G_{\mu\nu}(x) \equiv \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) = \sum_K^{all} m_K [\partial_\mu a_{K\nu}^-(x) - \partial_\nu a_{K\mu}^-(x)]_{x \neq \xi_K}$, and $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) = \sum_K^{all} q_K [\partial_\mu a_{K\nu}^+(x) - \partial_\nu a_{K\mu}^+(x)]_{x \neq \xi_K}$.

The vortexless partial solutions, $W_{N\mu\nu}^N(x) \equiv g_N^{\mu\rho} g_N^{\mu\lambda} W_{\rho\lambda}^N = 0$ and $W_{\mu\nu}^N(x) \equiv \partial_\mu P_{N\nu}(x) - \partial_\nu P_{N\mu}(x) = 0$, of the general equations (28)-(29) can be written through a gradient, $\partial_\mu \Upsilon_N(x)$, of the single valued scalar potential $\Upsilon_N(x)$, with $\partial_\mu \partial_\nu \Upsilon_N(x) = \partial_\nu \partial_\mu \Upsilon_N(x)$ and

$$m_N V_{N\mu}(x) = m_N \mathcal{V}_\mu + m_N B_\mu(x) + q_N A_\mu(x) + m_N \partial_\mu \phi_N(x) = m_N \partial_\mu \Upsilon_N(x). \quad (30)$$

One may associate this scalar $\Upsilon_N(x) = \int V_{N\mu} dx^\mu + const = s_N(x) + const$ with the proper four-interval s_N , because $d\Upsilon_N = dx^\mu \partial_\mu \Upsilon_N = dx^\mu V_{N\mu} = ds_N$. The potential microscopic states (30) correspond to the well known London supercurrent of Cooper's carriers in electromagnetic fields. For this reason the microscopic particle states may be considered as superfluid virtual fluctuations of the particle in its proper four-space. Vortexless motion of microscopic particle densities in vacuum can be read through the following three-vector functions

$$\begin{aligned} g_i(x) &\equiv \partial_o \mathcal{V}_i - \partial_i \mathcal{V}_o \equiv -\partial_o v_i \beta^{-1} - \partial_i \beta^{-1} \\ &= -G_i(x) - m_N^{-1} q_N E_i(x) \equiv \partial_o \beta^{-1} \sqrt{g_{oo}^N} g_i + \partial_i \beta^{-1} (\sqrt{g_{oo}^N} - 1), \end{aligned} \quad (31)$$

$$\begin{aligned} h^i(x) &\equiv -2^{-1} e^{ijk} (\partial_j \mathcal{V}_k - \partial_k \mathcal{V}_j) \equiv \{curl \beta^{-1} \mathbf{v}\}^i \\ &= -H^i(x) - m_N^{-1} q_N B_q^i(x) \equiv -\{curl \beta^{-1} \sqrt{g_{oo}^N} \mathbf{g}\}^i, \end{aligned} \quad (32)$$

where the universal tensor density $F_{\mu\nu}(x)$ forms the three-vector electric, $E_i(x) \equiv F_{oi}(x)$, and magnetic, $B_q^i(x) \equiv -e^{ijk} F_{jk}(x)/2\sqrt{\gamma}$ ($e^{123} = 1, \sqrt{\gamma} = 1$), fields, acting locally on the electric charge q_N of the extended vacuum state of the particle N. Gravity is implicated into (31) - (32) through the universal tensor $G_{\mu\nu}(x)$ based on potentials $B_\mu(x) = -G \sum_K m_K a_{K\mu}^-(x)_{x \neq \xi_K}$, with $G_i(x) \equiv G_{oi}(x) \equiv \partial_o B_i - \partial_i B_o$ and $H^i(x) \equiv -2^{-1} e^{ijk} G_{jk}(x) \equiv -2^{-1} e^{ijk} (\partial_j B_k - \partial_k B_j)$. We take that $\sqrt{\gamma} = 1$, $\beta = \sqrt{1 - \delta_{ij} v^i v^j}$, $\{curl \mathbf{a}\}^i \equiv (2\sqrt{\gamma})^{-1} e^{ijk} (\partial_j a_k - \partial_k a_j) = 2^{-1} e^{ijk} (\partial_j a_k - \partial_k a_j)$, and $div \mathbf{a} \equiv \gamma^{-1/2} \partial_i (\sqrt{\gamma} a^i) = \partial_i a^i$ for flat three-space. Notice directly from (31)-(32) in the absence of external fields that the elementary gravitational and electromagnetic self-fields are locally induced by the extended particle velocity $v_i(x)$ at vacuum points $x \neq \xi_N$.

The general equations (28)-(29) admit also non-potential microscopic states of the particle N with a finite vortex density, when $W_{\mu\nu}^N(x) \neq 0$. A number of elementary vortices crossing a 2D surface $s^{\mu\nu}$, $n = h^{-1} \int W_{\mu\nu}^N ds^{\mu\nu} = h^{-1} \int P_\mu^N dx^\mu$,

is to be an integer with respect to the Plank constant introduced by Wave Mechanics.

A natural question arise: why one observes in practice only point, localized particles and do not observes directly their extended microscopic states or virtual particle fluctuations in vacuum points? In order to answer this conceptual question we should find the stress-energy tensor density of these microscopic states.

5.4. The stress-energy tensor density of the elementary particle-field object

By varying the action (24) with respect to $\delta g_{\mu\nu}^N(x)$ one may find the stress-energy tensor density of elementary matter at all four-space points, including the particle vertex ξ_N . The self-field is not induced at the vertex point, where the localized, macroscopic particle state exhibits a nonzero energy-tensor, $T_N^{\mu\nu}(\xi_N[s_N]) \equiv m_N V_N^\mu(\xi_N[s_N]) V_N^\nu(\xi_N[s_N]) \neq 0$. The localized particle is not screened by its self-field at the vertex point, where the bare particle is observed in practice. The macroscopic equation of free motion, $\nabla_\mu T_N^{\mu\nu}(\xi_N) = m_N [V_\nu \nabla_\mu V^\mu(\xi_N) + D V_\nu(\xi_N)] = 0$, of the localized particle is consistent with its mass/charge four-currents conservation and with the macroscopic geodesic equation (22).

In vacuum points, $x \neq \xi_N, \xi_K$, we fix the four-vector dx_N^μ in $dx_{N\mu}(x) = g_{\mu\nu}^N(x) dx_N^\nu$ and $V_\mu^N(x) = dx_{N\mu}(x) / [dx_{N\nu}(x) dx_N^\nu]^{1/2}$ under the variations $\delta g_{\mu\nu}^N(x)$. Then the variation procedure should provide the symmetric stress-energy tensor density, $T_N^{\mu\nu}(x)_{x \neq \xi}$, of the elementary particle-field matter. Note that symmetric components of $g_{\mu\nu}$ are not independent one from another, $\delta g_{\mu\nu} = \delta g_{\nu\mu}$, and we define $\delta S \equiv - \int dx^4 \sqrt{-g} (T^{\mu\nu} \delta g_{\mu\nu} + T^{\nu\mu} \delta g_{\nu\mu}) / 2 - \delta \int dx^4 \sqrt{-g} r / 16\pi G$. When $i^\mu = \text{const}$, than $\delta(i^\mu V_\mu) = i^\mu \delta[g_{\mu\nu} dx^\nu (g_{\rho\lambda} dx^\rho dx^\lambda)^{-1/2}] = [(\delta g_{\mu\nu}) i^\mu dx^\nu / 2 ds] + [(\delta g_{\nu\mu}) i^\nu dx^\mu / 2 ds] = (i^\mu dx^\nu + i^\nu dx^\mu) \delta g_{\mu\nu} / 2 ds$. The term $-\sqrt{-g} (f_N^{\mu\nu} \delta W_{N\mu\nu} + f_N^{\nu\mu} \delta W_{N\nu\mu}) / 16\pi$ may be transformed into $(\sqrt{-g} \nabla_\nu f_N^{\nu\mu} / 4\pi) \delta V_{N\mu}$ under the integral. The contravariant metric tensor is related to the covariant one, *i.e.* $\delta g^{\alpha\beta} = -g^{\alpha\mu} g^{\beta\nu} \delta g_{\mu\nu} - g^{\alpha\nu} g^{\beta\mu} \delta g_{\nu\mu} = -\delta g_{\mu\nu} (g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu})$, $\delta \sqrt{-g} = \sqrt{-g} (g^{\mu\nu} \delta g_{\mu\nu} + g^{\nu\mu} \delta g_{\nu\mu}) / 2 = \sqrt{-g} (g^{\mu\nu} + g^{\nu\mu}) \delta g_{\mu\nu} / 2$, with $\sqrt{-g} = \sqrt{g_{00}}$.

Finally, after the variation of (24) at the vertex free points with respect to $\delta g_{\mu\nu}^N$ (and $\delta g_{\nu\mu}^N$), one can derive an Einstein-type equation,

$$\frac{1}{8\pi G} \left(r_N^{ik}(x) - \frac{1}{2} g_N^{\mu\nu} r_N(x) \right) = T_N^{\mu\nu}(x) \equiv \frac{P_N^\mu(x) I_N^\nu(x) + P_N^\nu(x) I_N^\mu(x)}{2} + \frac{W_{\rho\lambda}^N(x)}{16\pi} [g_N^{\mu\nu} f_N^{\rho\lambda}(x) - 2g_N^{\mu\rho} f_N^{\nu\lambda}(x) - 2g_N^{\nu\rho} f_N^{\mu\lambda}(x)], \quad (33)$$

with the stress-energy tensor density $T_N^{\mu\nu}(x) \equiv T_N^{\mu\nu}(x)_{x \neq \xi_N, \xi_K}$ of the microscopic particle states and their self fields at all vacuum points $x \neq \xi_N, \xi_K$.

A trace of the stress-energy tensor density of elementary particle-field matter in (33) always vanishes in vacuum points, $g_N^{\mu\nu} T_N^{\mu\nu} \equiv 0$. This means from (33) that the scalar curvature of the proper four-space is also absent, $r_N(x) \equiv 0$. Recall that the Rainich - Misner criterion for unified theories [21,22] dismisses scalar curvature terms in the initial dynamical equations. The Ricci metric curvature, $R^{\mu\nu}(x)$, will be introduced below for the macroscopic Einstein equation after averaging the microscopic states over the global ensemble of matter.

By taking into account the vector variational equation (25), one may rewrite

the tensor equation (33) in the following form,

$$r^{ik}(x) = (\partial_\rho V_\lambda - \partial_\lambda V_\rho)[g_N^{\mu\rho}(-Gm_N f_N^{\nu\lambda}) + g_N^{\nu\rho}(-Gm_N f_N^{\mu\lambda}) - 2^{-1}g_N^{\mu\nu}(-Gm_N f_N^{\rho\lambda})]. \quad (34)$$

From here the traceless tensor curvature $r_N^{\mu\nu}(x)$ arises only for microscopic states with finite vorticity density, $\partial_\rho V_\lambda(x) \neq \partial_\lambda V_\rho(x)$, that maintains the tensor rank 2 of the Einstein equation.

The variations of (24) with respect to the field co-ordinate of elementary matter, δx_N^μ , lead to its equation of motion in all vacuum points $x \neq \xi_N, \xi_K$,

$$\begin{aligned} 2\nabla_\mu^N g_{\rho\nu}^N T_N^{\mu\rho} &= \frac{\nabla_\rho^N f_N^{\rho\lambda}}{4\pi} W_{\lambda\nu}^N + \frac{\nabla_\rho^N W_N^{\rho\lambda}}{4\pi} f_{\lambda\nu}^N = i_N^\lambda W_{\lambda\nu}^N \\ &= m_N w_{\mu\nu}^N(x) i_N^\mu(x) + [m_N G_{\mu\nu}(x) + q_N F_{\mu\nu}(x)] \frac{\nabla_\rho^N f_N^{\rho\mu}(x)}{4\pi} = 0, \end{aligned} \quad (35)$$

where we used variational equations (25), (28), and equalities $W^{\rho\lambda} \nabla_\nu f_{\rho\lambda} \equiv 2W^{\rho\lambda} \nabla_\rho f_{\nu\lambda}$, $f^{\rho\lambda} \nabla_\nu W_{\rho\lambda} \equiv 2f^{\rho\lambda} \nabla_\rho W_{\nu\lambda}$, $\nabla_\mu^N g_{\rho\nu}^N r_N^{\mu\rho} \equiv 2^{-1} \partial r_N = 0$. By contracting (35) with i_N^ν one finds an equality, $i_N^\nu i_N^\lambda W_{\lambda\nu}^N = 0$, *i.e.* there are only three independent equations in this four-component conservation law.

In fact, (35) is a geodesic equation for microscopic particle states in vacuum points $x \neq \xi_N, \xi_K$. By integrating $i_N^\lambda(x) W_{\lambda\nu}^N(x) = 0$ over all space points, one can derive the macroscopic geodesic equation (22) for the averaged, localized particle state. A self-force $f_\nu^N(x) = (q_N^2 - Gm_N^2) f_{\nu\mu}^N(x) \nabla_\rho^N f_N^{\rho\mu}(x) / 4\pi = -\nabla_\mu^N \Theta_{\nu N}^\mu(x)$, with $\Theta_{\nu N}^\mu(x) \equiv (q_N^2 - Gm_N^2)[4f_N^{\mu\rho}(x) f_{\rho\nu}^N(x) + \delta_\nu^\mu f_N^{\rho\lambda}(x) f_{\rho\lambda}^N(x)]$, in the equation (35) is related locally to the plain vorticity $w_{\nu\mu}^N$ of the microscopic particle state in vacuum. This vacuum self-action on the extended particle states is relevant to dynamics of the averaged, localized particle in the macroscopic equation (22). The total space integral of the vacuum self-force $f_N^\mu(x)$ is responsible for the observed radiation damping.

In closing we may conclude from (26) and (33) that the microscopic mass and charge four-currents and the trace of the microscopic stress-energy tensor density of every elementary particle and its self fields are balances strictly to zero at all vacuum points. The point vertex ξ_N is the only unscreened mass/charge/energy carrier, which can directly be observed in practice.

6. The global superposition of microscopic states

Even though the covariant equations are four-dimensional in the proper four-space x_N^μ , dynamics of matter depends on the development parameter, and there must be a three-dimensional picture as seen by an observer. This motivates us to pick out the coordinate world time $x^o = x_N^o$ and the absolute time interval $dt = |dx^o| > 0$ for the evolution of matter at all points of the world three-space \mathbf{x} .

Recall that we operate in (25) with the microscopic four-flow density $i_N^\mu(x)$ of virtual particle fluctuations at all vacuum points $x \neq \xi_N, \xi_K$,

$$\begin{aligned} \int \frac{dx^\nu}{dp} \frac{\delta^4(x - \xi_N[p])}{\sqrt{-g}} dp &\equiv \int \frac{dx^\nu}{d\xi_N^o[p]} \frac{\delta^3(\mathbf{x} - \xi_N[p])}{\sqrt{\gamma}} \frac{\delta(x^o - \xi_N^o[p])}{\sqrt{g_{oo}^N}} d\xi_N^o[p] \\ &\equiv i_N^\nu(x) = \begin{cases} i_N^i(x) = \delta^3(\mathbf{x} - \xi_N[x^o]) dx^i / \sqrt{g_{oo}^N(x)} dx^o \\ i_N^o(x) = \delta^3(\mathbf{x} - \xi_N[x^o]) / \sqrt{g_{oo}^N(x)} \end{cases}. \end{aligned} \quad (36)$$

This four-flow density of the extended particle is locally bound with its microscopic fields. A formal transition from the virtual, microscopic states to the point particle and its retarded/advanced macroscopic fields, available for measurements, changes the structure of the particle flow density, $i_N^i(\xi) = \delta^3(\mathbf{x} - \xi_N[x^o])d\xi^i/\sqrt{g_{oo}^N(\xi)}dx^o$ and $i_N^o(\xi) = \delta^3(\mathbf{x} - \xi_N[x^o])/\sqrt{g_{oo}^N(\xi)}$ in the Maxwell-type equations (25)-(25a). Then the macroscopic electromagnetic and gravitational self fields exhibit retarded, $s_N(x, \xi_N[\tau_+]) = 0$, and advanced, $s_N(x, \xi_N[\tau_-]) = 0$, zero four-interval behaviour with respect to the point particle at ξ_N . Formally one may say that these fields with the opposite evolution directions are induced by outgoing ($dx^o > 0$) and incoming ($dx^o < 0$) virtual particle fluctuations. There are no local intersections of point classical particles, $\xi_N \neq \xi_K$. But there is a global intersection of their extended microscopic states. Therefore one may derive microscopic dynamic equations with local intersections of the extended particle states and read these equations in terms of averaged, macroscopic functions for point particles and retarded/advanced fields.

Every considered world space point \mathbf{x} can be related to vertices of different nonlocal particles by zero-interval conditions. In other words, extended states of different particles can cross one common world space point $\mathbf{x} \equiv \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N, \dots$ like light of distant stars crosses the Earth at any fixed time. Intersection of microscopic states of all nonlocal particle-field objects in one common (world) 3D space \mathbf{x} may be described under one common co-ordinate time $x^o(\mathbf{x})$, which is independent from proper parameters of different objects. All proper three-spaces x_K^i , associated with different particles K, have universal 3D metrics with $\gamma_{ij}^K = \delta_{ij}$, due to (15). For this reason the flat coordinate space \mathbf{x} with the universal 3D interval $dl = \sqrt{\delta_{ij}dx^i dx^j} > 0$ and the flat co-ordinate time with the absolute 1D interval $dt = |dx^o(\mathbf{x})| = \sqrt{\delta_{oo}dx^o dx^o} > 0$, are quite appropriate to apply to all matter. Recall that the four-interval $|ds_N(x)| = \sqrt{g_{\mu\nu}^N dx^\mu dx^\nu} > 0$ depends on local fields and, consequently, on the path parameters p_N of elementary particles.

One may say that the world space \mathbf{x} is a flat 3D projection of the infinite kaleidoscope of curved nonlocal 4D particle-field objects, while $|dx^o(\mathbf{x})|$ is simply an absolute world rate for stage set changes on this invisible 3D screen. Every vacuum point of the world space \mathbf{x} at any particular moment $x^o(\mathbf{x})$ may contain encountered intersections of microscopic particle-field states with zero mass/charge/energy densities. We write this statement by summing the microscopic equations (26) and (33) over the global intersection of all elementary objects. This leads to the following microscopic equations and their macroscopic generalizations for the total mass four-current density (respectively for virtual, nonlocal particles and point, classical particles associated with a selected vacuum point $x \neq \xi_N, \xi_K, \dots$),

$$\sum_N^{all} m_N \sqrt{g_{oo}^N} i_N^\nu(x)_{x \neq \xi_N} = -\frac{1}{4\pi G} \sum_N^{all} \partial_\mu (-G m_N) \sqrt{g_{oo}^N} f_N^{\mu\nu}(x)_{x \neq \xi_N} \quad (37)$$

$$\mu_o(\mathbf{x}) \frac{dx^\nu}{\sqrt{g_{oo}dx^o}} = -\frac{1}{4\pi G \sqrt{g_{oo}}} \partial_\mu \sqrt{g_{oo}} G^{\mu\nu}(x), \quad (37a)$$

for the total electric four-current density,

$$\sum_N^{all} q_N \sqrt{g_{oo}^N} i_N^\nu(x)_{x \neq \xi_N} = \frac{1}{4\pi} \sum_N^{all} \partial_\mu q_N \sqrt{g_{oo}^N} f_N^{\mu\nu}(x)_{x \neq \xi_N}, \quad (38)$$

$$\rho_o(\mathbf{x}) \frac{dx^\nu}{\sqrt{\tilde{g}_{oo}} dx^o} = \frac{1}{4\pi\sqrt{\tilde{g}_{oo}}} \partial_\mu \sqrt{\tilde{g}_{oo}} F_N^{\mu\nu}(x), \quad (38a)$$

and for the total stress-energy tensor density,

$$\begin{aligned} & \sum_N^{all} \frac{m_N \sqrt{\tilde{g}_{oo}^N}}{2} (i_N^\mu(x) V^\nu + i_N^\nu(x) V^\mu)_{x_N \neq \xi_N} \\ & \equiv \sum_N^{all} \left\{ \delta^3(\mathbf{x} - \xi_N) m_N \frac{dx^\mu}{dx^o} \frac{dx^\nu}{ds_N} \right\}_{\mathbf{x} \neq \xi_N} = \frac{1}{8\pi G} \sum_N^{all} \sqrt{\tilde{g}_{oo}^N} r_N^{\mu\nu}(x)_{x_N \neq \xi_N} \\ & - \frac{1}{8\pi G} \sum_N^{all} \left[-G m_N \frac{dx^\mu}{ds_N} \sqrt{\tilde{g}_{oo}^N} \nabla_\rho^N f_N^{\rho\nu}(x) - G m_N \frac{dx^\nu}{ds_N} \sqrt{\tilde{g}_{oo}^N} \nabla_\rho^N f_N^{\rho\mu}(x) \right. \\ & \quad \left. - G m_N M_{\rho\lambda}^N(x) \sqrt{\tilde{g}_{oo}^N} [g_N^{\mu\rho} f_N^{\nu\lambda}(x) + g_N^{\nu\rho} f_N^{\mu\lambda}(x) - \frac{g_N^{\mu\nu}}{2} f_N^{\rho\lambda}(x)] \right]_{x \neq \xi_N} \\ & + \frac{F_{\rho\lambda}(x)}{8\pi} \sum_N^{all} \left\{ q_N \sqrt{\tilde{g}_{oo}^N} [g_N^{\mu\rho} f_N^{\nu\lambda}(x) + g_N^{\nu\rho} f_N^{\mu\lambda}(x) - \frac{g_N^{\mu\nu}}{2} f_N^{\rho\lambda}(x)] \right\}_{x \neq \xi_N}, \quad (39) \end{aligned}$$

$$\begin{aligned} & \mu_o(\mathbf{x}) \frac{dx^\mu}{\sqrt{\tilde{g}_{oo}} dx^o} \frac{dx^\nu}{ds} = \frac{1}{8\pi G} \left(R^{\mu\nu}(x) - \frac{\tilde{g}^{\mu\nu}}{2} \tilde{g}_{\rho\lambda} R^{\rho\lambda}(x) \right) \\ & - \frac{1}{16\pi} [2\tilde{g}^{\mu\rho} F_{\rho\lambda}(x) \tilde{F}^{\lambda\nu}(x) + 2\tilde{g}^{\nu\rho} F_{\rho\lambda}(x) \tilde{F}^{\lambda\mu}(x) + \tilde{g}^{\mu\nu} F_{\rho\lambda}(x) \tilde{F}^{\rho\lambda}(x)]. \quad (39a) \end{aligned}$$

Here we introduced global compositions of elementary macroscopic (advanced/retarded) fields, $G^{\mu\nu}(x) \equiv [\tilde{g}_{oo}(x)]^{-1/2} \sum_N^{all} (-G m_N) \sqrt{\tilde{g}_{oo}^N} f_N^{\mu\nu}(x)_{x \neq \xi_N}^{s_N[\tau_-]=0}$, $F_{\rho\lambda}(x) \equiv \sum_N^{all} q_N f_{\rho\lambda}^N(x)_{x \neq \xi_N}^{s_N[\tau_+]=0}$, $\sqrt{\tilde{g}_{oo}(x)} F^{\mu\nu}(x) \equiv \sum_N^{all} \sqrt{\tilde{g}_{oo}^N} q_N f_N^{\mu\nu}(x)_{x \neq \xi_N}^{s_N[\tau_+]=0}$, and $\sqrt{\tilde{g}_{oo}(x)} \tilde{F}^{\mu\nu}(x) \equiv \sum_N^{all} \sqrt{\tilde{g}_{oo}^N} g_N^{\mu\nu}(x) q_N f_N^{\rho\lambda}(x)_{x \neq \xi_N}^{s_N[\tau_+]=0}$ in order to use them in the macroscopic equations at vacuum points $x \neq \xi_N$. Notice that all elementary microscopic fields contribute to these macroscopic fields. This is not a case for the macroscopic particle densities $\mu_o(x)$ and $\rho_o(x)$ in vacuum (field) points, because the macroscopic particle state (resulting from averaging over the microscopic states under small macroscopic time scales) is localized at one point ξ_N . The global sums of the microscopic densities $\sum_N^{all} m_N \delta^3(\mathbf{x} - \xi_N)$ and $\sum_N^{all} q_N \delta^3(\mathbf{x} - \xi_N)$ at the field point \mathbf{x} are to be replaced under macroscopic time with a summing over those particles, which are localized in a small macroscopic volume $\Omega \rightarrow 0$ around \mathbf{x} , $\sum_N^{all} m_N \delta^3(\mathbf{x} - \xi_N) \rightarrow \mu_o(\mathbf{x}) \equiv \sum_N^\Omega m_N(\xi_N)/\Omega$, and $\sum_N^{all} q_N \delta^3(\mathbf{x} - \xi_N) \rightarrow \rho_o(\mathbf{x}) \equiv \sum_N^\Omega q_N(\xi_N)/\Omega$.

The macroscopic metric tensor $\tilde{g}_{\mu\nu}(x)$ for an elementary ensemble of all point particles in Ω is defined in the field, vacuum point $x \neq \xi_N$ by (15) with an elementary mass $m_N = \mu_o(\mathbf{x})\Omega$ and an elementary charge $q_N = \rho_o(\mathbf{x})\Omega$. Space is flat for macroscopic gravitation and electrodynamics, because $\tilde{g}_{oi}\tilde{g}_{oj}\tilde{g}_{oo}^{-1} - \tilde{g}_{ij} = \delta_{ij}$, $\tilde{\gamma} = 1$, and $\sqrt{-\tilde{g}} \equiv \sqrt{\tilde{\gamma}\tilde{g}_{oo}} = \sqrt{\tilde{g}_{oo}}$.

It is worth to repeat that all point sources-outlets are excluded from the original microscopic equations (37), (38), and (39) for vacuum points. Contrary to the continuous density of the microscopic, extended particle states, the density of the macroscopic, point particles in one selected field point \mathbf{x} is meaningless. One should not neglect this obvious fact under interpretation of the field equations for electrodynamics or gravitation.

The macroscopic Ricci tensor $R^{\mu\nu}(x) \equiv \mathcal{G}^{\mu\nu}(x) - 2^{-1}\tilde{g}^{\mu\nu}\tilde{g}_{\rho\lambda}\mathcal{G}^{\rho\lambda}(x)$ in (39a) and the Einstein tensor $\mathcal{G}^{\mu\nu}(x)$ are defined by a following superposition of the elementary traceless tensors $r_N^{\mu\nu}$ and advanced gravitational fields,

$$\mathcal{G}^{\mu\nu}(x) \equiv \frac{1}{\sqrt{\tilde{g}_{oo}}} \sum_N^{all} \sqrt{g_{oo}^N} \left(r_N^{\mu\nu} + Gm_N \left[\frac{dx^\mu}{ds_N} \nabla_\rho^N f_N^{\rho\nu}(x) + \frac{dx^\nu}{ds_N} \nabla_\rho^N f_N^{\rho\mu}(x) \right] \right. \\ \left. + Gm_N M_{\rho\lambda}^N(x) [g_N^{\mu\rho} f_N^{\nu\lambda}(x) + g_N^{\nu\rho} f_N^{\mu\lambda}(x) - \frac{g_N^{\mu\nu}}{2} f_N^{\rho\lambda}(x)] \right) \Big|_{x \neq \xi_N[\tau-]}^{s_N[\tau-]=0}. \quad (40)$$

The scalar Ricci "curvature", $\tilde{g}_{\rho\lambda}R^{\rho\lambda}(x) \equiv R(x) = -\mathcal{G}(x) \equiv -\tilde{g}_{\rho\lambda}\mathcal{G}^{\rho\lambda}(x)$, originates from advanced, incoming elementary fields. Recall that these gravitational fields were induced in order to screen microscopic mass four-current densities of nonlocal particles in vacuum points, rather than to curve flat three-space. One may say the scalar "curvature" R of macroscopic fields balances the mass density $\mu_o(\mathbf{x})$ of macroscopic particles in vacuum points. This "curvature" may be found from the macroscopic gravitational equation (39a) or directly from the original microscopic equality $\sum_N \sqrt{g_{oo}^N} g_{\mu\nu}^N [8\pi GT_N^{\mu\nu}(x) - r_N^{\mu\nu}(x)]_{x \neq \xi_N, \xi_K} \equiv 0$,

$$R(x) = \frac{-8\pi G \mu_o(\mathbf{x}) d\tilde{s}}{\sqrt{\tilde{g}_{oo}} dx^o} = \frac{8\pi}{\sqrt{\tilde{g}_{oo}}} \sum_N^{all} (-Gm_N) \frac{g_{\mu\nu}^N dx^\mu dx^\nu}{ds_N dx^o} \delta_N^3(\mathbf{x} - \xi_N). \quad (41)$$

One may find from (14) a global sum for the four-momentum densities, $\mathcal{P}_\mu^{mic}(x) \equiv \sum_N^{all} \delta^3(\mathbf{x} - \xi_N) m_N g_{\mu\nu}^N dx^\nu / ds_N$, of all microscopic states in any vacuum point and a macroscopic four-momentum density, $\mathcal{P}_\mu^{mac}(x) \equiv \mu_o(\mathbf{x}) \tilde{g}_{\mu\nu} dx^\nu / d\tilde{s}$, of point particles in the nearest vicinity of the selected vacuum point $x = \{\mathbf{x}; x^o\} \neq \xi_N, \xi_K$,

$$\mathcal{P}_\mu^{mic}(x) = \sum_N^{all} m_N \mathcal{V}_{N\mu}(x) \delta^3(\mathbf{x} - \xi_N) + B_\mu(x) \sum_N^{all} m_N \delta^3(\mathbf{x} - \xi_N) \\ + A_\mu(x) \sum_N^{all} q_N \delta^3(\mathbf{x} - \xi_N) + \sum_N^{all} m_N \delta^3(\mathbf{x} - \xi_N) \partial_\mu \phi_N, \quad (42)$$

$$\mathcal{P}_\mu^{mac}(x) = \mu_o(\mathbf{x}) [\tilde{\mathcal{V}}_\mu + B_\mu(x)] + \rho_o(\mathbf{x}) A_\mu(x) + \mu_o(\mathbf{x}) \partial_\mu \tilde{\phi}_N(x). \quad (42a)$$

The local field potentials $A_\mu(x)$ and $B_\mu(x)$ contribute with the identical signs to the macroscopic four-momentum density (42a), but with the opposite signs to (37a) and (38a) for the mass and charge four-current densities, respectively. By considering these macroscopic relations one finds that the electric current densities can screen locally external electromagnetic fields, while the mass current densities can be responsible only for oscillating solutions of external gravitational fields without their damping. In other words there are no screens in practice for macroscopic gravitational fields. The physical reason may be traced as follows: the point charge can interact with casual electromagnetic waves, while advanced disturbances of the incoming gravitational field never arrive from infinity in finite times and "inform" the point mass in the laboratory.

The requirement for finite magnitudes for all material densities supports our introduction of the extended particle states in vacuum. It seems very unlikely that it is possible to overcome the problem of divergence in electrodynamics,

for example, by applying the classical paradigm of the point charge to microscopic field theory. The developed microscopic approach to gravitation is quite consistent with the cited Einstein's statements, which worth to be repeated: "A coherent field theory requires that all its elements be continuous ... And from this requirement arises the fact that the material particle has no place as a basic concept in a field theory. Thus, even apart from the fact that it does not include gravitation, Maxwell's theory cannot be considered as a complete theory" (translation [19]). Our dual formalism for the macroscopic/microscopic particle within the extended elementary object is a predicted way to introduce the "continuous element" into the classical field equations for electrodynamics and gravitation.

7. Conclusion

The particle mass four-current $m_K i_K^\mu(x)$ is the origin of the incoming gravitational field $(-Gm_K f_{\mu\nu}^{\kappa-})$ and the vector gravitational force $m_N(-Gm_K)V_N^\nu f_{\mu\nu}^{\kappa-}$. According to General Relativity the electromagnetic and gravitational parts of interactions cannot compensate each other in the state of general motion due to the different tensor nature of these interactions. The electromagnetic and gravitational external potentials for the charged particle have a unified retarded/advanced structure, $\sum_{K \neq N}^{\kappa \neq N} (q_N q_K a_{K\mu}^+ - Gm_N m_K a_{K\mu}^-)$, in our approach, which lead the vector balance of electromagnetic and gravitational "instantaneous" forces for a two-body system with $q_1 q_2 = Gm_1 m_2$.

The absence of aberrations of the "instantaneous" Newton and Coulomb forces, based on the advanced and retarded field potentials, respectively, does not contradict to the finite speed c of gravitational and electromagnetic waves. Could Sun's incoming gravitational wave come to the laboratory from infinity, it would have an advanced aberration 20 arc seconds with respect to the attraction force direction and 40 arc seconds with respect to the retarded aberration of Sun's light.

The developed coupling of electromagnetic and gravitational vector interactions and the integration of the particle into its field structure satisfy the predicted double unified criterion [19], as well as the Rainich-Misner known criterion [21,22] for the unified field theory. All point sources-outlets are excluded from the continuous particle-field equations in agreement with Einstein's approach [23,19,20] to the continuum theory, and all physical magnitudes in vector electrogravity are free from divergences.

We accepted only retarded potentials for outgoing electromagnetic fields with respect to their point sources and only advanced potentials for incoming gravitational fields with respect to their point outlets. The opposite directions of outgoing and incoming spherical fields explain the repulsion of identical charges and the attraction of masses in the vector field theory. The unified vector nature of electrogravity corresponds to the unified spin-1 photon/graviton approach to electromagnetic/gravitational waves. Our theory predicts the absence of metric modulations of flat world space in disagreement with the tensor (spin-2) gravitational wave propounded by General Relativity.

The electromagnetic/gravitation waves have zero charge/mass four-current densities. These pure field objects do not couple to each other, contrary to particles with finite four-currents. The vector gravitational wave, *i.e.* the electromagnetic antiwave in our approach, may be formally associated with a negative-energy hole in the retarded radiation spectrum due to the inverse time

rate for the advanced, incoming field matter. The electromagnetic repulsion of identical charges is mediated by the virtual particles (photons) in quantum field theory, where the gravitational attraction of masses may be formally mediated by holes (gravitons) in the retarded field spectrum. This approach creates a clear perspective to quantize the gravitational spin-1 field in a full analogy with the successfully quantized electromagnetic field.

Based on electrodynamics and its references, vector gravitation becomes a self-contained theory, which may be applied to practice without references on other gravitomechanical theories. Nevertheless, the developed approach to gravitation coincides with the Newton theory in the non-relativistic limit and with the concept of flat world space x^i and the absolute time rate $dt(x^i) = dt(x^j)$ in all space points. The linear relativistic corrections to Newton's motion in the present theory with flat three-space coincide quantitatively with the similar corrections of General Relativity, but our nonlinear solutions for strong fields do not lead to GR's black holes. The available observations of all known kinds of interactions and all conservation laws do not contradict to Euclidean geometry of the world space, which we considered as a material continuum of the microscopic particle states and their fields with vanishing charge/mass/energy densities at all space points except at point sources-outlets.

The main relativity tests (the gravitation light bending by the Sun, the radar echo delay, and the Mercury perihelion precession) can be quantitatively explained under flat three-space with the absolute time rate in all space points. The superfluid microscopic states of the nonlocal particle in vacuum are consistent with the Aharonov-Bohm phenomenon [24]. The nearly isotropic microwave background radiation corresponds to the spatial flatness of the Universe [3] that is in an agreement with the metric tensor (15) for elementary matter.

One may ask what is the antimass in the present theory with incoming gravitational fields. A spontaneous split of the zero-mass "object" should create at least a couple of mass-antimass outlets with their advanced fields, which moves separately toward these outlets. The electric charge source and its antisource attract each other due to their outgoing fields and tend to collapse, while the mass outlet and anti-outlet repulse each other due to their incoming fields and tend to disintegrate. The repulsion of galaxies with the mass and the anti-mass should lead to an increase of their mutual velocities and to the Universe expansion with acceleration.

The dual approach to the microscopic/macrosopic particle and its fields in (25)-(25a), for example, is based on their instantaneous ($C/c=\infty$) virtual fluctuations with zero energy tensor density within the nonlocal particle-field object. This is quite consistent with instantaneous reshaping of Newton/Coulomb forces and generation of advanced macroscopic waves in infinity. These infinitely distant gravitational waves have the light speed c , like the retarded electromagnetic waves, and will never reach the laboratory in finite time intervals. Therefore the advanced gravitation wave do not violate the casual requirements of Special Relativity for macroscopic fields and particles. Nonetheless, such generation-at-a-distance of gravitation waves "there" (in infinity) requires energy consumption by accelerated masses "here" and leads, for example, to radiation damping for rotating binary pulsars. More general, the unified approach to elementary matter through the nonlocal particle-field object N , which is self-enclosed into infinite four-space x_N^μ , satisfies to the Mach's principle - matter there (gravitational waves) governs inertia (of point particles) here [25, 8]. Anyway, the nonlocal nature of the Universe, confirmed last century in different tunnel and

light experiments, for example [26, 27], is open for further exploration and discussions.

References

- [1] Einstein A 1916 *Annalen der Physik* **49** 769
- [2] Schwarzschild K 1916 *Kl. Math.-Phys. Tech.* (Sitzungber. Deut. Akad. Wiss., Berlin) 424,
- [3] De Bernardis P *et al.* 2000 *Nature* **404** 955, Lange A *et al.* 2001 *Phys. Rev. D* **63** 042001, Hanany S *et al.* 2000 *Astrophys. J.* **545** L5
- [4]. Rosen N 1940 *Phys. Rev.* **57** 147, Logunov A A 1995 *Physics - Uspekhi* **38** 179
- [5] Landau L D and Lifshitz E M 1971 *The Classical Theory of Fields* (Oxford: Pergamon)
- [6] Weinberg S 1972 *Gravitation and Cosmology* (New York: John Wiley and Sons)
- [7] Wald R M 1984 *General Relativity* (Chicago: The University of Chicago Press)
- [8] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (Freeman, San Francisco: Freeman)
- [9] Will C M *Theory and Experiment in Gravitational Physics* 1981 (Cambridge: Cambridge University Press)
- [10] Synge J L 1960 *Relativity: the General Theory* (Amsterdam: North-Holland Publishing Company)
- [11] Einstein A 1907 *Jahrb. f. Radioaktivität u. Elektronik* **4** 411
- [12] Einstein A 1911 *Annal. der Phys.* (Leipzig) **35** 898
- [13] Pound R V and Rebka G A 1960 *Phys. Rev. Lett.* **4** 337
- [14] Shapiro I I 1964 *Phys. Rev. Lett.* **13** 789
- [15] Williams J G *et. al.* *Phys. Rev. Lett.* **36** 551
- [16] Shapiro I I, Counselman C C and King R W 1976 *Phys. Rev. Lett.* **36** 555
- [17] Hilbert D 1917 *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen* **4** 21
- [18] De Broglie L 1923 *Compt. Rend. Acad. Sci. Paris* **174**, 506, 548, 630; de Broglie L 1973 *Wave Mechanics: the First Fifty Years* eds W C Price, S S Chissick and T Ravensdale (London: Univer. of London King's College)
- [19] Tonnelat M - A 1966 *The Principles of Electromagnetic Theory and Relativity* (Dordrecht: Reidel Publishing Co)
- [20] De Broglie L 1962 *New Perspectives in Physics* (New York: Basic Books)
- [21] Rainich G Y 1925 *Trans. Am. Math. Soc.* **27** 106
- [22] Misner C and Wheeler J 1957 *Ann. of Physics* **2** 525
- [23] Einstein A 1956 *The Meaning of Relativity* (Princeton: Princeton University Press) Appendix II
- [24] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
- [25] Mach E 1904 *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt* (Leipzig: Brockhaus F A) p 236
- [26]. Aspect A, Grangier P and Roger G 1981 *Phys. Rev. Lett.* **47** 460
- [27]. Tittel W, Brendel J, Zbinden H and Gisin N 1998 *Phys. Rev. Lett.* **81** 3563